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Discount rates for use in calculating the J-value

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Summary

The Life Quality Index (LQI) and the J-value use discount rates in two contexts: (i) to discount the utility of the individual's future earnings and (ii) to discount costs and benefits from schemes that have societal benefit. The former is termed the "net discount rate" while the latter is termed the "social discount rate", and used to convert the stream of costs that is reasonable for a protection scheme at a J-value of unity into a single, up-front figure. The net discount rate is shown to depend on the three parameters: the pure time discount rate, the nation's growth rate, and the average person's risk-aversion. An appeal is then made to Ramsey's economic model, and a derivation of Ramsey's result is given, which links the social discount rate to the same three parameters. It is then established that the net discount rate is equal to the social discount rate minus the rate of growth of Gross Domestic Product. The difficulties associated with the subjectivity of the pure time discount rate are discussed, but these may be bypassed in the UK context if the Treasury-recommended value for the social discount rate is accepted. Accepting the Treasury's view on the UK's average growth rate also then enables the net discount rate to be found by subtraction.

Different countries will experience different conditions, particularly for growth rate. Nevertheless the mathematical framework developed in this paper may be used to estimate appropriate figures for those countries.

Key words: Life Quality Index, J-value, social discount rate, net discount rate, pure time discount rate.

1. Introduction

The J-value method calculates the Judgement- or J-value as the ratio of the actual cost of a safety system to the maximum sensible expenditure found using the Life Quality Index (LQI). The LQI and the J-value use discount rates in two contexts:

1. to discount the utility of the individual's future earnings
2. to discount costs and benefits from schemes that have societal benefit.

The former is termed the "net discount rate", *r*. The latter is termed the "social discount rate", *r**, and used to convert the stream of costs that is reasonable for a protection scheme at $J = 1$ into a single, up-front figure.

Appropriate figures for discount rates have been discussed in Nathwani, Lind and Pandey (2008) for a number of countries. This paper concentrates on the approach used in the UK, which is based on the mathematical method devised by Ramsey in 1928. A derivation of Ramsey's result will be given which links the social discount rate to the three parameters: the pure time discount rate, λ, the nation's growth rate, *g*, and the average person's risk-aversion, ε. The net discount rate is shown to depend on the same governing parameters and hence direct link is established between the social discount rate and the net discount rate.

There are problems in measuring the pure time discount rate, for while economists feel that it should be non-negative, some data series would suggest a violation of that constraint. Hence a subjective view is used by economists in the UK and North America. In the context of the UK, this results in a Treasury-recommended value for the social discount rate. The UK Treasury also recommends an average growth rate for the UK economy, and it is possible to compute the net discount rate from the difference between the two figures. The result is two discount rates for using the J-value in the UK that correspond with officially endorsed figures.

Different countries will experience different conditions, particularly on growth rate. Nevertheless the mathematical framework given in this paper may be used to calculate discount rates for those countries.

2. The social discount rate, *r**, and the pure time discount rate, λ

The ethical position is taken with the J-value and LQI is that the next hour of life for every member of the public should be valued the same whether the individual is rich or poor, young or old. As a result, the J-value method incorporates the default assumption that an equal share of the nation's gross domestic product (GDP, £year^{-1}) will be distributed to all individuals in the nation (Thomas *et al.*, 2006a). The laws of economics suggest that the individual will value successive increments of this income differently – the initial cash needed to sustain life and limb will be seen as vital, but later additions will be regarded as less important, although assuredly still welcome. Economists account for this differential valuation by assuming that it is the utility of income that matters to the individual, rather than the money itself. Utility, *u*, is then held to be a continuous function of income, *c*, *viz.* $u(c)$, normally with decreasing slope (Pratt, 1964, Thomas, 2010). Moreover, it is standard economic practice (Ramsey, 1928) to discount the utility of income being received in future years.

Using the Power utility function, explained and justified in Thomas (2010) as the most realistic utility function, the average person may be regarded as having enjoying a utility, u_0, from his current income, c_0, given by:

$$u_0 = c_0^{1-\varepsilon} \qquad (1)$$

where the risk-aversion, ε, has been found from diverse, inferential methods, to lie between 0.82 and 0.85 in the UK (Thomas, Jones and Kearns, 2010, Thomas *et al.*, 2010 a, b). The average value, 0.835 is very close to the value ε = 0.83 recommended by Pearce and Ulph (1995). It will be assumed for simplicity that ε is constant over time. Thus an expression equivalent to equation (1) will apply for the utility in a future year, *k*, with the subscript, 0, being replaced by the subscript, *k*.

The Power Utility of equation (1), which will be valid when $0 \le \varepsilon < 1$, constitutes a ratio scale (Thomas, 2010). This means that it is correct to a positive multiplier in the same way as absolute temperature, where, for example, the Kelvin and Rankine scales of measurement, differing by a multiplier of 1.8, are equally valid.

The J-value takes the average person's total utility from income from now on to be the sum of his current and future utilities, discounted using the pure time discount rate (also known as the time-preference rate) for utility, λ (year^{-1}):

$$u = \sum_{k=0}^{K-1} e^{-\lambda k} c_k^{1-\varepsilon} \qquad (2)$$

where *K* is the average person's life expectancy (years), assumed discrete in this formulation (it is about 41 years in the UK). It is assumed that λ is constant over time. (The symbol, r_d, has been used for λ in Thomas *et al.* (2006): $r_d = \lambda$).

The average person can expect his income to grow over time, and the growth rate inherent in this model is the rate of the growth of GDP. This is reasonable, since if GDP per head grows, then society as a whole can be expected to be better off. We will assume that the growth rate will be constant over time at an average value, *g*. Hence:

$$c_k = e^{gk} c_0 \qquad \text{for } 0 \le k \le K-1 \qquad (3)$$

Substituting from equation (3) into equation (2) gives:

$$\begin{aligned} u &= \sum_{k=0}^{K-1} e^{-\lambda k} \left(c_0 e^{gk}\right)^{1-\varepsilon} \\ &= c_0^{1-\varepsilon} \sum_{k=0}^{K-1} e^{-\lambda k} e^{g(1-\varepsilon)k} \qquad (4) \\ &= c_0^{1-\varepsilon} \sum_{k=0}^{K-1} e^{-(\lambda - g(1-\varepsilon))k} \end{aligned}$$

The composite expression, $\lambda - g(1-\varepsilon)$, may be named the "net discount rate", *r*, the term introduced in Thomas *et al.* (2006), Appendix A:

$$r = \lambda - g(1-\varepsilon) \qquad (5)$$

The net discount rate accounts for growth in earnings over time.

Equation (4) may be re-expressed in the form of a summation of the discounted utilities for each year, u_k:

$$u = u_0 + u_1 + \ldots + u_k + \ldots + u_{K-1} \qquad (6)$$

where

$$u_k = e^{-(\lambda - g(1-\varepsilon))k} c_0^{1-\varepsilon} \qquad (7)$$

Given that λ, g and ε are taken to be constant, the only way that the total utility, *u*, can change in this model is for c_0 to change. A change in c_0 will alter each of the annual incomes, c_k, via equation (3), which will lead, in turn to an alteration in each of the discounted utilities, u_k. Hence, from equation (6), a small change in year 0 income, Δc_0, will give rise to a change in total utility, Δu:

$$\begin{aligned}\Delta u &\approx \frac{du}{dc_0}\Delta c_0 \\ &= \left(\frac{du_0}{dc_0} + \frac{du_1}{dc_0} + \ldots + \frac{du_k}{dc_0} + \ldots + \frac{du_{K-1}}{dc_0}\right)\Delta c_0 \quad (8) \\ &= \Delta u_0 + \Delta u_1 + \ldots + \Delta u_k + \ldots + \Delta u_{K-1}\end{aligned}$$

where a formal differentiation of equation (6) gives the 2nd line, while Δu_k, given by:

$$\Delta u_k = \frac{du_k}{dc_0}\Delta c_0 \qquad (9)$$

may be regarded as the contribution to the change in total utility from each year.

Meanwhile differentiation of equation (7), gives the differential, du_k / dc_0:

$$\frac{du_k}{dc_0} = (1-\varepsilon) e^{-(\lambda - g(1-\varepsilon))k} c_0^{1-\varepsilon} \qquad (10)$$

Now the change in income in year, *k*, Δc_k, will be related to Δc_0 by:

$$\Delta c_0 \approx \frac{dc_0}{dc_k}\Delta c_k \qquad (11)$$

But from equation (3),

$$c_0 = e^{-gk} c_k \tag{12}$$

Hence the differential, dc_0 / dc_k , will be:

$$\frac{dc_0}{dc_k} = e^{-gk} \tag{13}$$

Thus combining equations (9), (10), (11) and (13) gives:

$$\begin{aligned}
\Delta u_k &\approx \frac{du_k}{dc_0} \frac{dc_0}{dc_k} \Delta c_k \\
&= (1-\varepsilon) e^{-(\lambda - g(1-\varepsilon))k} e^{-gk} c_0^{1-\varepsilon} \Delta c_k \\
&= (1-\varepsilon) e^{-(\lambda - g + g\varepsilon + g)k} c_0^{1-\varepsilon} \Delta c_k \\
&= (1-\varepsilon) e^{-(\lambda + g\varepsilon)k} c_0^{1-\varepsilon} \Delta c_k
\end{aligned} \tag{14}$$

Let us posit a small increment in income in year *k*, Δc_k , given by:

$$\Delta c_k = \Delta c^* \qquad \text{for any } k\text{: } 0 \le k \le K - 1 \tag{15}$$

If this increase should occur in year 0, when *k* = 0, it will lead to a year 0 contribution to the change in total utility of

$$\Delta u_0 \approx (1-\varepsilon) c_0^{1-\varepsilon} \Delta c^* \tag{16}$$

If, however, the same increment in income were to be experienced in a later year $m,\ m > 0$, it would lead to a year *m* contribution to the change in total utility of

$$\Delta u_m \approx (1-\varepsilon) e^{-(\lambda + g\varepsilon)m} c_0^{1-\varepsilon} \Delta c^* \qquad m > 0 \tag{17}$$

Hence an increment in income experienced in year $m,\ m > 0$ will lead a reduced contribution to the change in total utility compared with the same increment in income experienced in year 0, with the ratio given by:

$$\frac{\Delta u_m}{\Delta u_0} \approx e^{-(\lambda + g\varepsilon)m} \qquad m > 0 \tag{18}$$

Equation (18) shows that the contribution to the change in total utility of the increment in income in year $m,\ m > 0$ is reduced below the contribution from year 0 by a discount factor of $e^{-(\lambda + g\varepsilon)m}$, which implies an effective discount rate of $\lambda + g\varepsilon$. As noted previously, λ is the (pure time) discount rate applicable to future utility, while $g\varepsilon$ accounts for growth in future income.

A household will be prepared in practice to defer consumption by saving if the savings' interest rate, r^*, is equal to $\lambda + g\varepsilon$:

$$r^* = \lambda + g\varepsilon \qquad (19)$$

Equation (19) matches the equation derived first by Ramsey (1928).

The rate, $r*$, also serves as a social discount rate, in that utility accruing in the future would be discounted using this rate. As pointed out by Stern (2009), pp. 81 – 82, a higher value of risk-aversion, ε, would imply that the wealth of the typical person of the future, who will be richer, should be discounted more heavily. The higher risk-aversion would result in a higher discount rate, reducing the importance of the greater wealth expected in the future. The social discount rate, $r*$, is recommended by the Treasury as the "standard real discount rate" (H. M. Treasury, 2003, 2011) when comparing costs and benefits of schemes that have societal benefit. Protection against human harm clearly comes within this remit.

Let us now consider the components of equation (19), starting with the term, $g\varepsilon$. The risk-aversion, ε, may be taken to vary sufficiently slowly with time to be represented well by a constant value, but the growth in GDP varies significantly over time, as shown in Figure 1 (ONS, 2011a), which illustrates its behaviour over more than 60 years. The average value of GDP growth, g (% p.a.), is given by:

$$g = 2.41 \qquad \text{for 1949 to 2010} \qquad (20)$$

Data restricted to more recent periods give similar values:

$$g = 2.95 \qquad \text{for 1995 to 2007} \qquad (21)$$

$$g = 2.18 \qquad \text{for 1995 to 2010} \qquad (22)$$

Equation (21) gives the average value up to the significant financial turbulence of 2008, while equation (22) includes it. Taking the long-term average of equation (20) as representative of the UK economy, and using a risk-aversion of $\varepsilon = 0.835$, as discussed above, then

$$g\varepsilon = 2.01 \approx 2.0 \text{ \% p.a.} \qquad (23)$$

This parallels the figure of 2% p.a. given in H. M. Treasury (2003, 2011), Annex 6.

The time-preference rate, λ, is not observable directly, but equation (19) provides a conceptual framework for its estimation of λ as:

$$\lambda = r^* - g\varepsilon \qquad (24)$$

However, the real savings' interest rate available to households, r^*, varies rapidly, as shown in Figure 2 (Bank of England, 2011, ONS, 2011b). More significantly, the savings rate is heavily dependent on the central bank rate, which is used as a control mechanism for monetary policy, either by Government or from the late 1990s, by the Bank of England. Therefore it represents a necessarily distorted response to the market of savers.

If equation (24) were to be applied to UK data for 1995 to 2010, then the results would be as shown in Figure 3. The time-preference rate, λ, appears to vary significantly over time, sometimes quite wildly. Furthermore, its average value, λ_{ave}, is negative at –1.23. Perhaps surprisingly, it is even more negative if the financial turbulence from 2008 onwards is omitted from consideration: -1.46. A negative value of λ_{ave} would imply that the average UK citizen would usually prefer to defer satisfaction: he would prefer to have "jam tomorrow" than today. This contradicts both standard economic theory and common observation.

As a result of this problem, two approaches are possible. In the first, an attempt is made to estimate or else posit a representative value of the household savings interest rate, r^*, and then use equation (24) to find a representative time-preference rate, λ. For example, setting $r^* = 5\%$ would suggest a pure time preference rate, $\lambda = 5 - 2 = 3\%$, while setting $r^* = 3.5\%$ would suggest a pure time preference rate, $\lambda = 3.5 - 2 = 1.5\%$. In this context, the comment of Boadway and Bruce (1984) is illuminating:

> "In the end, the appropriate discount rate to use will reflect individual judgement"

as is the suggestion by Johansson (1991) in his Section 9.8, "Choice of discount rate":

> "In reality, there are many different interest rates, reflecting, among other things, capital market imperfections and the presence of complicated tax schemes. The problem faced by the cost-benefit practitioner is then to decide what discount rate to choose. The practical solution chosen by many evaluators is to calculate PVs [present values] for different discount rates."

after which he cites 3% p.a. and 12% p.a. as examples – clearly a very wide range.

The other approach is to attempt to estimate λ directly. The report, Oxera (2002), provides a wide-ranging review and discussion here, and recommends splitting the pure time-preference rate into:

$$\lambda = \delta + L \qquad (25)$$

where δ is the time preference rate conditional on the person surviving to enjoy the utility, while L is the probability of dying in the coming year, equal to the fraction of the population who will die in the coming year.

Equation (25) is stated without derivation in Oxera (2002), but a rationale for the formula may be developed as follows. Let n_1 be the number of people who will die in the coming year, while the remaining $n - n_1$ will survive the year. Let those who will survive adopt a pure time preference rate, δ. But those expecting to die within a year would set their value of discount rate for future years very high indeed, since they would expect not to benefit from income in future years. Let their pure time preference rate be δ_1. The average value would then be:

$$n\lambda_{ave} = (n - n_1)\delta + n_1\delta_1 \qquad (26)$$

or

$$\begin{aligned}\lambda_{ave} &= \left(1 - \frac{n_1}{n}\right)\delta + \frac{n_1}{n}\delta_1 \\ &= (1 - L)\delta + L\delta_1\end{aligned} \qquad (27)$$

The percentage of people expected to die in the next year is about 1%, implying $L = n_1/n \approx 0.01$ = 1% p.a.. Hence $(1 - L) \approx 1.0$. A high value of δ_1 might be $\delta_1 = 1.0$, or 100% per year. An approximate argument to justify $\delta_1 = 1.0$ might be as follows. Viewed as an interest rate on his capital, $\delta_1 = 1.0$ would give the person expecting to die within the year an interest payment within the year (actually at the end) equal to his starting sum, even if his principal was withheld from him throughout the year and thus until after he was dead. It might be argued that he would then be indifferent between keeping his starting sum and spending it during the year and receiving the interest at rate, $\delta_1 = 1.0$, and spending that same amount of money during his last year of life. Employing $\delta_1 = 1.0$ gives:

$$\lambda_{ave} \approx \delta + L \qquad (28)$$

which compares directly with equation (25).

The fact that equation (25) may be derived from this simple model means that it may be open to the same criticisms that may be levelled against the model. The model presumes that either that the people who will die in the next year are aware of the fact and adopt $\delta_1 = 1.0$ as a result, or that, although they are unaware of their impending demise, the value $\delta_1 = 1.0$ is chosen for them, which would appear to be something of an approximation. Furthermore, a continuation of the reasoning would suggest that those who will die within 2 years should set a discount rate, δ_2, that, while not quite as high as δ_1, would be higher than δ. In fact, one would expect a sequence of discount rates dependent on when one expected to die: $\delta_1 > \delta_2 > \delta_3 > ...\delta_{76} > \delta_{77} > \delta_{78} > \delta_{79}$, with the last figure applying to someone just born, who will have a life expectancy of about 79 years. On this basis, the value of δ would be the weighted average: $\lambda_{ave} = \sum_{i=1}^{79} q_i\delta_i$, where q_i is the fraction of the population with a life expectancy of *i* years. The problem is then that an assessment would be needed for δ_i for all *i: i* = 1, 2, ..., 79.

In practice, it is accepted that it is very hard to observe δ, but the case is made in Oxera (2002) for a value of about $\delta \approx$ 0.5%. It is further suggested that a zero value for δ would imply that each generation should restrict its living standard to a subsistence level (just sufficient to allow for the birth and upbringing of the next generation). This paradox follows from the fact that the sum total of the utilities of the infinite number of future generations would be valued very much more highly than its own utility.

Thus Oxera (2002) recommends

$$\lambda = \delta + L \approx 1.5 \ \ \text{\% p.a.} \qquad (29)$$

which, when added to $g\varepsilon \approx 2.0$ gives a social discount rate for the UK of

$$r^* \approx 3.5 \ \ \text{\% p.a.} \qquad (30)$$

which is the base value recommended by H. M. Treasury (2003, 2011), taken to be valid for payment periods of 30 years or less.

On the other hand, based on his "own rough point-guesstimate of what most economists might think", based on a survey of 2000 economists, Weitzman (2007) suggests that $\lambda = 2.0$ % p.a., which would lead to a social discount rate of $r^* \approx 4\%$ per annum for the UK.

3. The social discount rate, *r**, and the net discount rate, *r*, to be used in J-value calculations

Equation (5) may be expanded to the form

$$r = \lambda + g\varepsilon - g \qquad (31)$$

Since $\varepsilon \approx 1$, it follows that the net discount rate is close to the pure time preference rate: $r \approx \lambda$. More precisely, we may use equation (19) to replace $\lambda + g\varepsilon$ by r^*, so that

$$r = r^* - g \qquad (32)$$

Hence if we decide to accept the values of social discount rate, $r^* = 3.5\%$ year^{-1} and average growth, $g = 2.0\%$ year^{-1} given in H.M. Treasury (2003, 2011), Annex 6, then it is unnecessary to investigate further the values of the pure time discount rate, λ, and the risk-aversion, ε. We may write immediately the net discount rate, *r*, to be used in calculating discounted life expectancy:

$$r = 3.5 - 2 = 1.5 \qquad \text{\% year}^{-1} \qquad (33)$$

Using UK actuarial data from 2009 and setting the risk-aversion to $\varepsilon = 0.82$, the average value of life to come is calculated to be £5.214 M when both the social discount rate and the net discount rate are set to zero.

When a social discount rate of 3.5% p.a. and a net discount rate of 1.5% p.a. are applied, the average value of life to come in the UK is calculated to be £2.274 M.

4. Conclusions

The principles behind the two discount rates used in the J-value, the social discount rate and the net discount rate, have been investigated using the Ramsey model. It has been shown how one component of the social discount rate can be derived from objective economic data, while the second component, the pure time preference rate, is considered to incorporate subjective judgement.

The Treasury provides a recommended, baseline value for the social discount rate, and it has been shown how the net discount rate can be found as the difference between this and the growth rate of the economy. A social discount rate of 3.5% per annum and a net discount rate of 1.5% per annum are recommended for use with the J-value in the UK. But although discount rates have been evaluated for the UK in particular, the mathematical framework is applicable generally, to all countries.

Finally, it may be recorded that in additional work not reported here, the basis for the Treasury's recommendation of a tapering social discount rate for long time periods (30+ years to 300+ years) has been explained. Moreover an investigation has been carried out into Lord Stern's fears that discounting has the effect of valuing the lives of future generations less than today's (Stern, 2007, 2009). With the J-value, the discounting is performed over the life of the individual at most, in both the J-value's modes of discounting. Thus the J-value's formulation avoids the concerns on discount rates put forward by Professor Weitzman (1998), Lord Stern and the Treasury.

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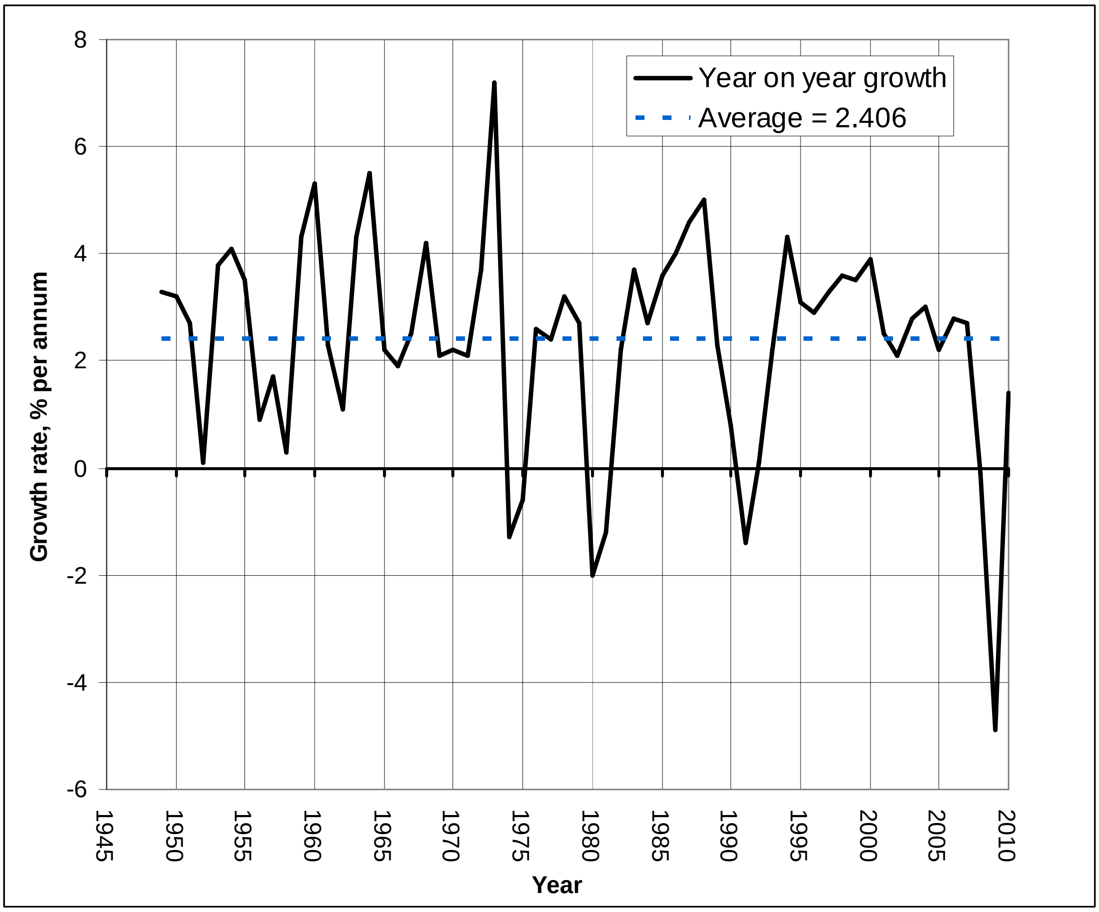

Figure 1 UK growth rate, 1949 to 2010 (ONS, 2011a)

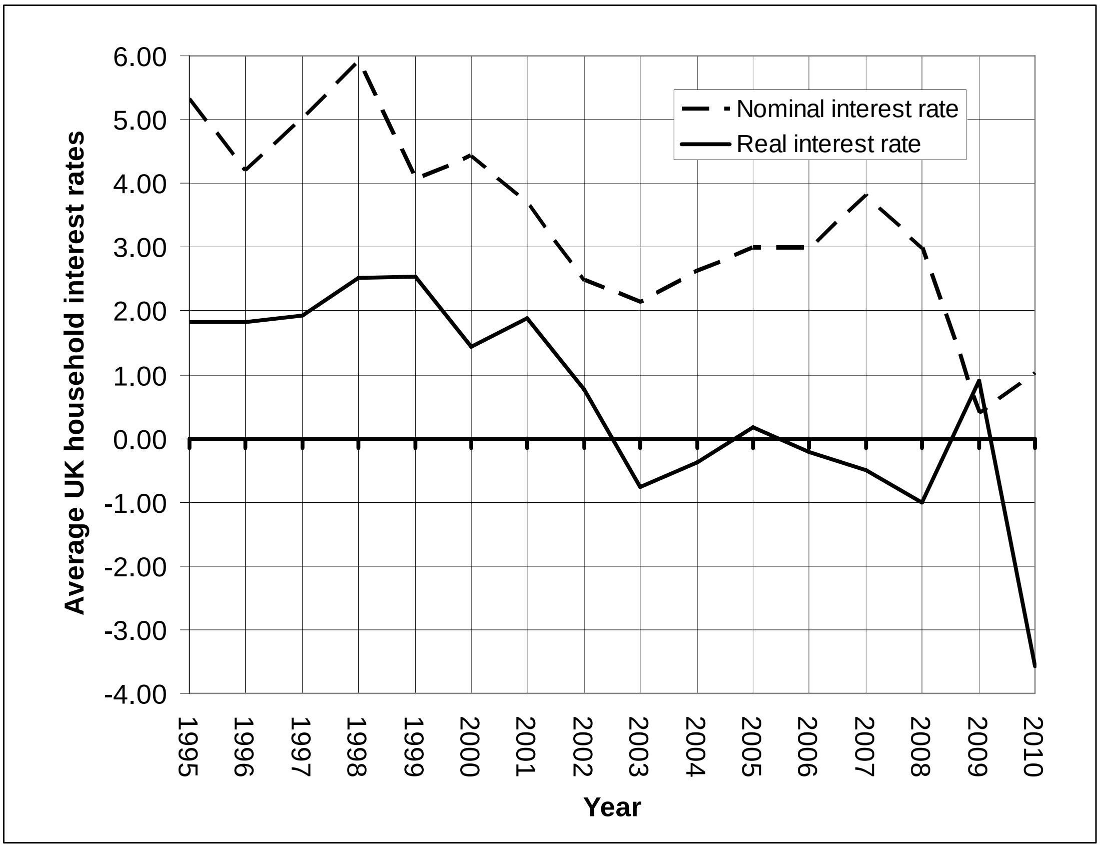

Figure 2 UK average real interest rates, 1995 to 2010 (Bank of England, 2011 and ONS, 2011b)

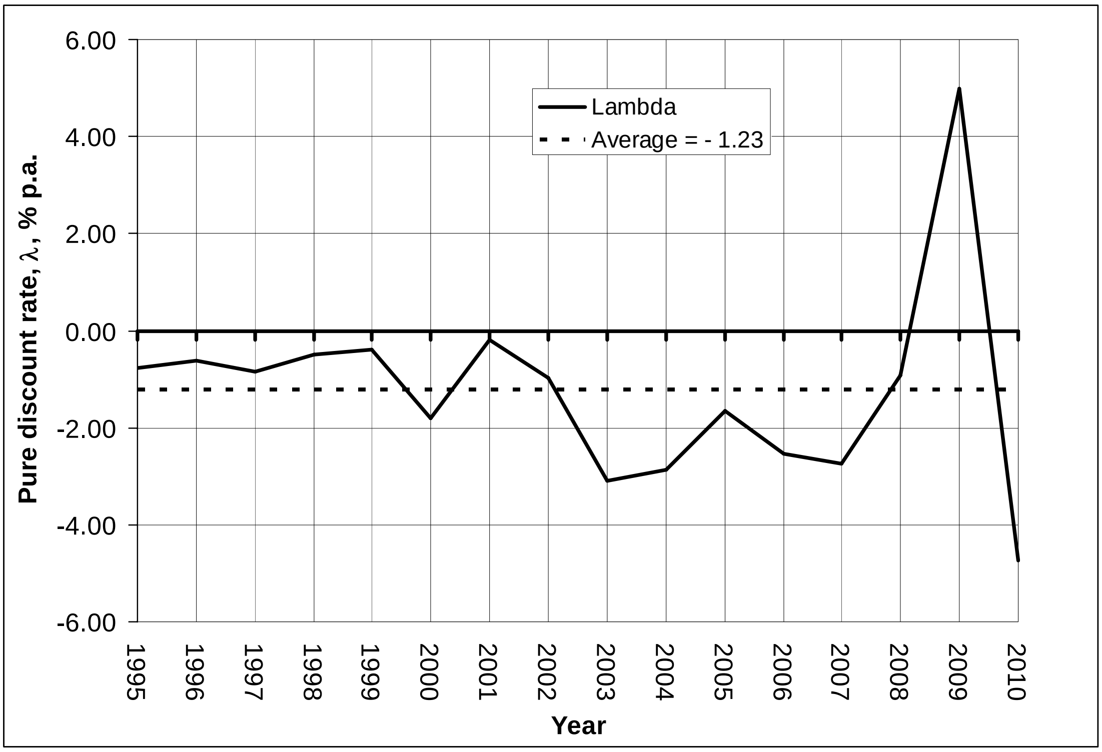

Figure 3 UK pure discount rate, λ, 1995 to 2010, as calculated from equation (20); average growth rate, g, over the period = 2.18 % p.a., ε = 0.835