Deriving target reliabilities from the LQI

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Abstract
A simple way to ensure cost efficiency during code-making or the probabilistic design of individual structures is to make use of the tentative target reliabilities provided in the JCSS Probabilistic Model Code. An acceptable level of life safety is, however, not guaranteed when relying on the JCSS target reliabilities, which have been derived based on monetary optimization. The goal of the present paper is to derive minimum target reliabilities from the Life Quality Index (LQI) that may be used in combination with the JCSS target reliabilities.

1. Introduction

The choice of a target reliability is an important first step for the calibration of structural design codes as well as during the probabilistic design of structures outside the code envelope. The Probabilistic Model Code [1] issued by the Joint Committee of Structural Safety (JCSS) provides tentative target reliabilities for different structural classes, see Table 1. The target reliabilities relate to structural system failure (or, in approximation, to the dominant failure mode on component level) and have been designed based on monetary optimization studies. The target reliabilities are given as a function of the costs of the risk reduction measure and the consequences in case of failure, both defined relative to the initial construction costs of the structure at hand. A generic approach relating the optimal reliability of structural systems to the safety costs and failure consequences was described by Rackwitz [2]. The target reliabilities in Table 1 were defined in consistency with the results of a parameter study performed by Rackwitz (see Annex B of [2]).

Relative cost of safety measure	**Consequences of failure**		
	Minor	**Moderate**	**Large**
Large (A)	$\beta = 3.1\left(P_f \approx 10^{-3}\right)$	$\beta = 3.3\left(P_f \approx 5\cdot 10^{-4}\right)$	$\beta = 3.7\left(P_f \approx 10^{-4}\right)$
Normal (B)	$\beta = 3.7\left(P_f \approx 10^{-4}\right)$	$\beta = 4.2\left(P_f \approx 10^{-5}\right)$	$\beta = 4.4\left(P_f \approx 5\cdot 10^{-6}\right)$
Small (C)	$\beta = 4.2\left(P_f \approx 10^{-5}\right)$	$\beta = 4.4\left(P_f \approx 5\cdot 10^{-6}\right)$	$\beta = 4.7\left(P_f \approx 10^{-6}\right)$

Table 1: Tentative target reliabilities related to one year reference period and ultimate limit states, based on monetary optimization, JCSS [1].

Human consequences of structural failure are discussed only qualitatively during the definition of consequence classes provided in the Probabilistic Model Code [1]. Risk to life can be taken into account quantitatively by introducing compensation costs for human fatalities in the consequence assessment. The social acceptability of the structural design in terms of risk to human life is, however, not necessarily guaranteed when relying on the JCSS target reliabilities. Figure 1 shows the interaction between monetary optimization and an acceptance criterion for risk to life. Within the acceptable region the optimization may be performed by a private or societal decision maker, but the acceptance criterion always has to be evaluated from a societal point of view. The acceptance threshold can be defined based on the marginal life saving costs principle, using the LQI net benefit criterion to judge the efficiency of life saving measures from a societal point of view. Only efficient investments into life safety have to be performed, but higher safety levels are of course also acceptable and may be aimed at if required by monetary optimization.

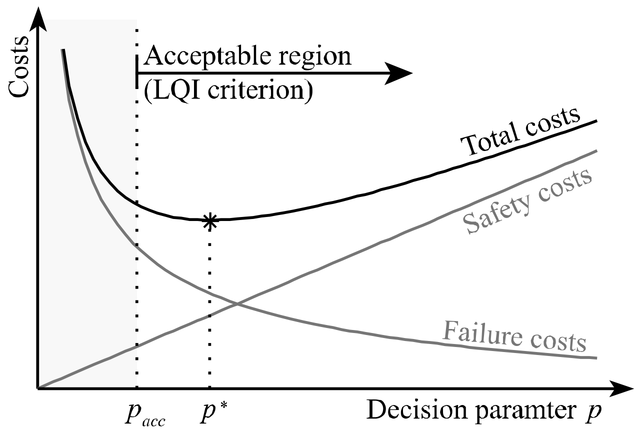

Figure 1: The LQI acceptance criterion as a boundary condition for monetary optimization.

The JCSS target reliabilities are derived from monetary optimization, assuming that the optimal design is also acceptable in terms of risk to life and limb. The goal of the present paper is to define the acceptance threshold in Figure 1 by deriving target reliabilities from the LQI criterion. They should be understood as minimum requirements that may be complemented by the target reliabilities provided in Table 1 if monetary optimization is not performed explicitly during the probabilistic design of a structure. The approach chosen for defining the LQI target reliabilities should thus be consistent with the monetary optimization studies performed by Rackwitz [2] forming the basis for the JCSS target reliabilities.

The content of the paper is outlined as follows: In section 2, a generic approach for defining target reliabilities based on the LQI is developed, using Rackwitz' formulation for monetary optimization as a starting point. The generic approach is then used to study the influence of different parameters on the LQI acceptance threshold, which is presented in section 3 together with a discussion of the interaction between the LQI criterion and monetary optimization. Finally, in section 4 a simple format is derived for defining minimum target reliabilities based on the LQI. We conclude with a short discussion of the strengths and limitations of the approach and open questions that may arise during the application of the results presented in this paper.

2. Generic formulation

The type of decision problem relevant for defining target reliabilities for structural systems can be described as follows: Investments into structural safety decrease the probability of an adverse event (termed "failure") that may lead to both monetary and human consequences. The goal is to define both the optimal level of safety (monetary optimization) and the minimum life safety requirements defining the acceptable region within which monetary optimization is admissible (LQI acceptance criterion). In the following we will first introduce the approach followed by Rackwitz [2] to determine the monetary optimum and then add the LQI criterion as a boundary condition imposed by society.

2.1 Monetary optimization

The approach followed by Rackwitz [2] was to define the objective function $Z(p)$ for monetary optimization based on renewal theory (Rosenblueth and Mendoza [3]):

$$\max_p \left\{ Z(p) = B - C(p) - A(p) - D(p) - U(p) \right\} \tag{1}$$

Here, B denotes the benefit derived from the existence of the structure, $C(p)$ the construction costs, $A(p)$ the obsolescence costs, $D(p)$ the ultimate limit state failure costs and $U(p)$ the serviceability failure costs. As the benefit B is assumed to be independent of the decision parameter p, it is clear that equation (1) is equivalent to minimizing the total costs. In the following, it will furthermore be assumed that the serviceability failure costs can be neglected (see Rackwitz [2] for the influence of serviceability costs on the results). This leads to the following simplified formulation:

$$\min_p \left\{ T(p) = C(p) + A(p) + D(p) \right\} \tag{2}$$

The obsolescence costs and failure costs are evaluated as expected values and discounted back to the time of the decision with a (private or public) discount rate γ in order to be comparable to the construction costs, which accrue at $t = 0$. For the estimation of the obsolescence costs it is assumed that structures are systematically renewed with a yearly obsolescence rate ω, such that

$$A(p) = C(p) \cdot \omega / \gamma \tag{3}$$

Demolition costs are assumed to be independent of the decision parameter p and can be neglected. The same renewal assumption is made for the case of structural failure, such that the damage costs $D(p)$ is estimated as follows:

$$D(p) = \left(C(p) + H \right) \cdot \lambda P_f(p) / \gamma \tag{4}$$

Here H denotes the costs that accrue in case of failure in addition to the costs of reconstruction, and λ is the intensity of a random disturbance process that may lead to failure with (conditional) probability $P_f(p)$. For ease of notation λ will be omitted in the following (assuming $\lambda = 1$), such that $P_f(p)$ refers to the yearly probability of failure or (unconditional) failure rate. Rackwitz estimated $P_f(p)$ from a simple limit state function with the decision parameter p being defined as the central safety factor:

$$P_f(p) = P[R - S < 0] \qquad p = E[R]/E[S] \tag{5}$$

Both the resistance R and the load effect S are modelled as random variables with coefficients of variation V_R and V_S, respectively. Finally the construction cost $C(p)$ is modelled as a linear function of the central safety factor p:

$$C(p) = C_0 + C_1 p \tag{6}$$

The objective function in equation (1) can now be reformulated as follows:

$$\max_{p}\left\{T\left(p\right)=\left[C_0+C_1 p\right]+\left[\left(C_0+C_1 p\right)\cdot\omega/\gamma\right]+\left[\left(C_0+C_1 p+H\right)\cdot P_f\left(p\right)/\gamma\right]\right\} \tag{7}$$

By relating all cost components to the "fixed" construction cost C_0, optimal reliabilities can be defined as a function of the safety costs and failure consequences. The cost classes in Table 1 are defined by the relative cost of safety measure C_1/C_0, the obsolescence rate ω and the coefficients of variation V_R and V_S. The classification of consequence classes is based on the ratio between the total failure costs $C_0+C_1 p+H$ and the (fixed) construction cost C_0.

The derivation of target reliabilities from monetary optimization is not in the focus of this paper. In the following it will be assumed that the values provided in Table 1 are consistent with the renewal theoretic formulation discussed above.

2.2 LQI acceptance criterion

For the formulation of the LQI acceptance criterion, the marginal life saving costs have to be extracted from the objective function (1). As discussed in Fischer et al. [4], the monetary benefit term B is not taken into account in the acceptance criterion. The same holds for the failure costs $D(p)$ and $U(p)$, because reductions in the expected value of these two cost components can be regarded to be a monetary benefit of increasing safety. For the LQI criterion it is therefore sufficient to quantify the marginal increase in construction and obsolescence costs and the respective change in risk to life as a function of the decision parameter p. The acceptable region can now be defined by the following inequality:

$$\frac{d\left(C(p)+A(p)\right)}{dp}\geq-\frac{d\mu(p)\cdot N_P}{dp}\cdot\frac{g}{q}C_x \tag{8}$$

Here, risk to life is defined as the expected number of fatalities $\mu(p)\cdot N_P$ (here estimated from the mortality rate $\mu(p)$ and the population size N_P). The Societal Willingness To Pay (SWTP) to save one additional life, $g/q\cdot C_x$, is derived from the LQI (g denoting the gross domestic product, q the leisure-labour trade-off exponent and C_x a demographic constant). Using the same assumptions as in section 2.1, the LQI criterion is evaluated as follows:

$$C_1\left(1+\omega/\gamma_S\right)\geq-\frac{1}{\gamma_S}\cdot\frac{g}{q}C_x\cdot N_F\cdot\frac{dP_f\left(p\right)}{dp} \tag{9}$$

Risk to life is now quantified in terms of the failure probability P_f and the expected number of fatalities given structural failure, N_F. As the LQI criterion is a boundary condition imposed by society, the societal discount rate γ_S has to be used.

Criteria similar to (8) and (9) have already been introduced in several papers by Rackwitz and Streicher [5-7] with some differences regarding discounting and the treatment of obsolescence costs. For combining it with the formulation developed for monetary optimization, Rackwitz and Streicher relate the LQI criterion to the "relative costs of safety measure" C_1/C_0 by making assumptions on the absolute value of the fixed construction costs C_0. These assumptions make their results scale dependent. In the following, a more generic approach will be developed by relating the safety costs to the SWTP for the life safety benefit that can be achieved by the investment. Rearranging (9) leads to the following criterion:

$$-\frac{dP_f\left(p\right)}{dp}\leq\frac{C_1\left(\gamma_S+\omega\right)}{g/q\cdot C_x\cdot N_F}=K_1 \tag{10}$$

The numerator on the right-hand side of the inequality, $C_1(\gamma_S+\omega)$, indicates how much the yearly safety cost rise with a unit increase in the global safety factor p (γ_S can be understood as financing cost and ω as the cost of rebuilding the structure in case of obsolescence). For the denominator the human consequences of structural failure N_F have been transformed into monetary units by multiplying with the SWTP to save one additional life. LQI target reliabilities can now be derived as a function of the constant K_1.

For a specific structure, the K_1 constant may of course be estimated using assumptions on the relative costs of safety measure, as defined in section 2.1 for the monetary optimization. Table 2 shows K_1 values for different relative costs of risk reduction measure (C_1/C_0) and failure consequences (N_F) for the example of two types of structures with different absolute values for the construction costs C_0. Table 2 (a) refers to a typical Swiss office building with construction costs $C_0 = 2'000\,CHF/m^2$; The expected number of fatalities N_F are estimated per m^2 floor area of the building. Table 2 (b) gives K_1 values for a structure (e.g. a bridge) with total construction costs of $C_0 = 10 Mio.CHF$; The number of fatalities N_F here has to be assessed for the whole structure. Further assumptions made during the calculation of the K_1 values for both types of structures are a yearly obsolescence rate ω of 2% and a societal interest rate γ_S of 3%. The SWTP per life saved, $g/q\cdot C_x$, is set to 5.1 Mio. CHF.

(a) Office Building, C_0 = 2’000 CHF/m^2			
N_F/m^2	C_1 / C_0		
	0.001	**0.01**	**0.1**
0.0001	2E-04	2E-03	2E-02
0.001	2E-05	2E-04	2E-03
0.01	2E-06	2E-05	2E-04
0.1	2E-07	2E-06	2E-05

(b) Bridge, C_0 = 10 Mio.CHF			
N_F	C_1 / C_0		
	0.001	**0.01**	**0.1**
0.1	1E-03	1E-02	1E-01
1	1E-04	1E-03	1E-02
10	1E-05	1E-04	1E-03
100	1E-06	1E-05	1E-04

Table 2: K_1 values for two types of Swiss structures with different construction costs C_0 as a function of the relative costs of safety measure and the human consequences in case of failure.

The K_1 values contain information on the safety costs, on the failure consequences and on the SWTP for life safety. The resulting LQI target reliabilities will further be influenced by the assumptions made regarding the distributions of the basic random variables in the limit state function, R and S. This will be discussed in the following section.

3. Parameter study for deriving target reliabilities from the LQI

Before defining minimum target reliabilities based on the LQI criterion (10), it has to be discussed how different parameter assumptions made during the assessment of acceptability will influence the result. In the following, we first discuss the influence of parameter variations with respect to the determination of the LQI acceptance threshold. In section 3.2, the interaction with monetary optimization will be investigated.

3.1 Parameters influencing the LQI acceptance criterion

As illustrated in Figure 2, the problem of quantifying the LQI acceptance threshold $P_{f,acc}$ can be divided into three main parts: Safety costs, failure consequences and limit state function. The assumptions made during the quantification of the marginal safety costs and the human consequences in case of failure are summarized in the constant K_1 introduced in section 2.2. The focus will therefore in the following be on the influence of the probabilistic models used in the limit state function (5) for estimating the failure probability.

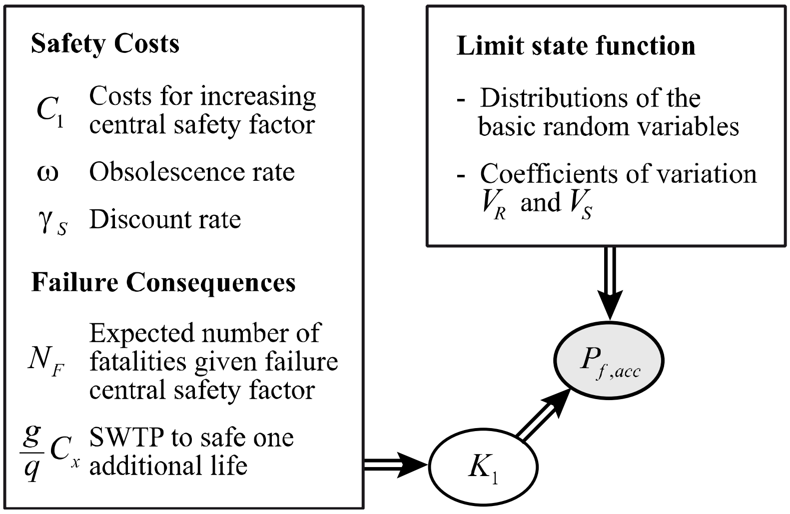

Figure 2: Parameters and assumptions influencing the LQI acceptance threshold.

In consistency with the numerical study by Rackwitz [2], it is first assumed that both the resistance R and the load effect S are lognormally distributed. Figure 3 (a) shows the LQI acceptance threshold $P_{f,acc}$ as a function of the constant K_1. A straight line on a log-log scale represents a power law. The special case of a linear dependency between $P_{f,acc}$ and K_1 can be assumed only for coefficients of variation V_R and V_S smaller than 0.3. This becomes clear in Figure 3 (b) where the ratio $P_{f,acc}/K_1$ is plotted as a function of K_1.

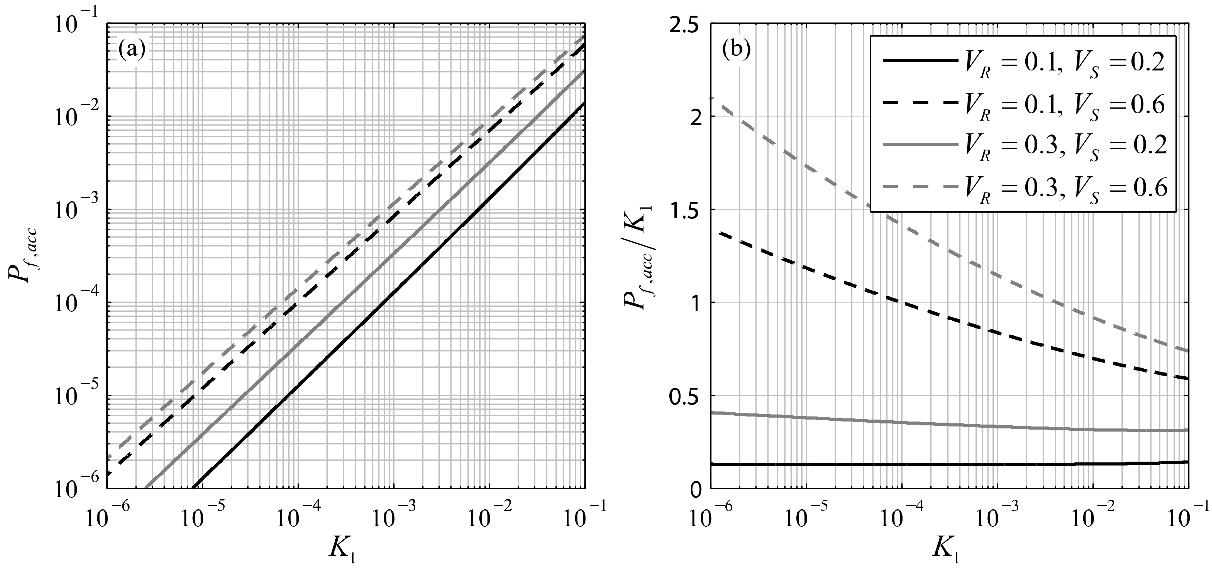

Figure 3: LQI threshold as a function of the constant K_1 for different coefficients of variation of the resistance R and the load effect S (both random variables lognormally distributed).

Figure 3 furthermore shows that the acceptable failure probability is higher for large coefficients of variation V_R and V_S. This can be explained by the fact that it is more costly to reduce the probability of failure if the variability of the basic random variables R and S is high. Therefore, large coefficients of variation have a similar effect as large safety costs C_1 (which in turn leads to a larger constant K_1). The required safety level grows with the efficiency of investments made for increasing safety. The influence of the coefficients of variation on the LQI acceptance threshold $P_{f,acc}$ is further investigated in Figure 4. The influence of one coefficient of variation (V_R or V_S) on the acceptance threshold is largest if the variability of the other random variable is small.

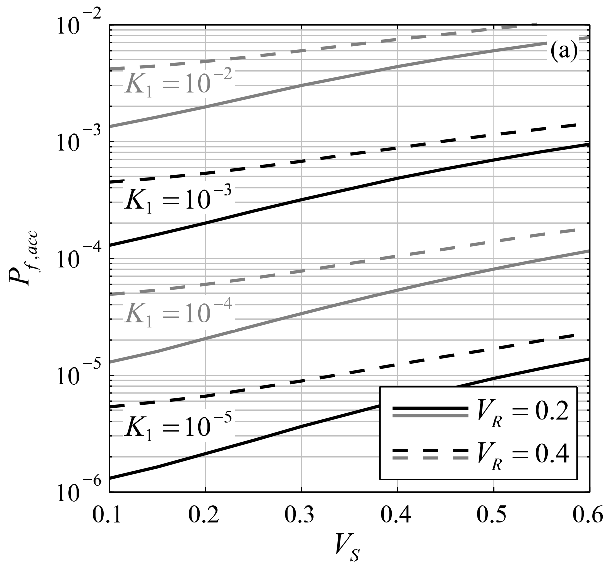

Figure 4: LQI acceptance threshold as a function of the coefficients of variation of the resistance R and the load effect S (both random variables lognormally distributed).

The results presented in Figure 3 and Figure 4 are valid for lognormally distributed random variables R and S. Figure 5 shows the effect of relaxing this assumption for the load effect S (Figure 5 (a)) or the resistance R (Figure 5 (b)). Black curves refer to a situation with small, grey curves to large coefficients of variation. It can be seen that the influence of distributional assumptions for the load effect S is relatively small when compared to the effect of assumptions regarding the coefficients of variation or the constant K_1. The same holds for the distribution of the resistance R as long as the variability of the basic random variables is small. However, in situations with large variability, the distributional assumptions for the resistance R can have a large impact (especially the coefficient of variation for the resistance, V_R, is important here).

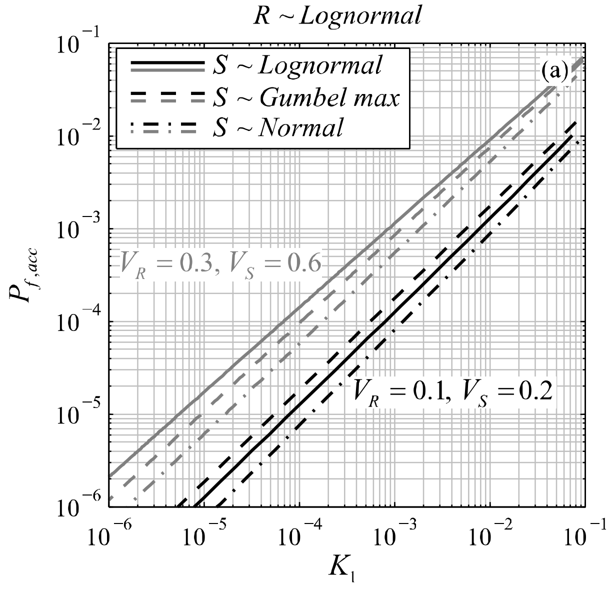

Figure 5: Influence of distributional assumptions (a) for the load effect S (with R lognormally distributed) and (b) for the resistance R (with S lognormally distributed).

Despite the inevitable differences caused by assumptions regarding the limit state function (5), a general trend can be observed regarding the behaviour of the LQI acceptance threshold $P_{f,acc}$ as a function of the constant K_1: Increasing K_1 by an order of magnitude leads roughly to an order of magnitude increase in the failure probability deemed to be acceptable by the LQI. This observation is valid in a broad range of situations and can therefore be used to derive a simple LQI target reliability format. This will be discussed in section 4. In section 3.2, the focus is on the interaction of the LQI criterion with monetary optimization.

3.2 Interaction with monetary optimization

As discussed already in the introduction, the LQI criterion defines the acceptable region within which monetary optimization is admissible (see Figure 1). A higher safety level than defined by the LQI target reliability should be aimed at if this is preferable from a monetary optimization point of view. The JCSS target reliabilities (Table 1) may be used as a substitute for direct optimization. In the following, a simple rule will be derived for checking whether the monetary optimum is an acceptable decision according to the LQI.

A necessary condition for the optimal design p^* is that the first derivative of the objective function has to be equal to zero at p^*. Using the objective function (2), the following condition can be derived ($dT\left(p^*\right)/dp = 0$):

$$\frac{dC\left(p^*\right)}{dp}\cdot\left(1+\omega/\gamma\right)+\frac{dC\left(p^*\right)}{dp}\cdot P_f\left(p^*\right)/\gamma+\frac{dP_f\left(p^*\right)}{dp}\cdot\left(C\left(p^*\right)+H\right)/\gamma = 0 \quad (11)$$

The second term can be neglected because $P_f\left(p^*\right) \ll \gamma$. Rearranging leads to the following equality:

$$\frac{dC\left(p^*\right)}{dp} = -\frac{C\left(p^*\right)+H}{\gamma+\omega}\cdot\frac{dP_f\left(p^*\right)}{dp} \quad (12)$$

Inserting (12) into the LQI acceptance criterion (8) gives the following result (note that $dP_f\left(p\right)/dp < 0$ for all p):

$$\frac{g/q\cdot C_x\cdot N_F}{C\left(p^*\right)+H} \leq \frac{\gamma_S+\omega}{\gamma+\omega} \quad (13)$$

Criterion (13) can be used for checking whether the monetary optimum p^* is an acceptable decision from a societal (life saving) point of view. It is valid for all decisions regarding investments that aim at decreasing the probability of an adverse event leading to both monetary and human consequences. The denominator on both sides of the inequality is evaluated from the point of view of the (private or societal) decision-maker while the numerator always quantifies societal preferences for life safety.

The failure costs H in equation (13) may include human compensation costs $H_C N_F$, with $H_C > 0$ denoting the compensation per fatality paid by the decision-maker (e.g. the owner of the structure). By rearranging and multiplying with $H_C N_F$, equation (13) can be reformulated as follows:

$$\frac{N_F H_C}{C\left(p^*\right)+H} \leq \frac{\gamma_S+\omega}{\gamma+\omega}\cdot\frac{H_C}{g/q\cdot C_x} \quad (14)$$

Criterion (14) is defined in terms of three ratios: The first term is the ratio between human compensation costs in case of failure, $N_F H_C$, and the total failure costs $C\left(p^*\right)+H$. This ratio is always smaller than one. The second ratio is a function of the societal discount rate γ_S and the discount rate γ that is used by the decision-maker performing the monetary optimization. In case of a societal decision-maker both the optimization and the LQI acceptance criterion are evaluated from a societal point of view, such that $\gamma = \gamma_S$ and $(\gamma_S + \omega)/(\gamma + \omega) = 1$. For private decision-makers the ratio typically becomes smaller than one because $\gamma > \gamma_S$. Finally, the third ratio relates to how loss of life is transformed into monetary terms for monetary optimization (compensation costs H_C) and for the LQI acceptance criterion (SWTP to safe one life $g/q \cdot C_x$), respectively.

The amount of human compensation costs H_C depends on a number of different factors. For a private decision-maker the amount paid per fatality is typically defined in a court sentence or based on negotiations with the relatives of the victim. Therefore, the compensation costs for a private decision-maker can vary considerably. In addition, the range of compensations paid depends on the legal system of a country. Also in the case of a societal decision-maker performing the optimization, there is no general consensus on how to quantify and include human compensation costs in monetary optimization. Several authors proposed formulas for quantifying H_C based on the LQI, using different terminology (see e.g. Lentz [8], Rackwitz [9] or Faber [10], among others). From the values proposed it is not even clear whether H_C should be smaller, equal to or larger than the SWTP $g/q \cdot C_x$ used for the LQI acceptance criterion. An extensive review on the quantification of societal human compensation costs for monetary optimization is beyond the scope of this paper. Also for the case of a private decision-maker, it is impossible to fix one value for H_C without limiting the investigation to one specific situation. Therefore the ratio $H_C/(g/q \cdot C_x)$ will be treated as a variable during the evaluation of criterion (14).

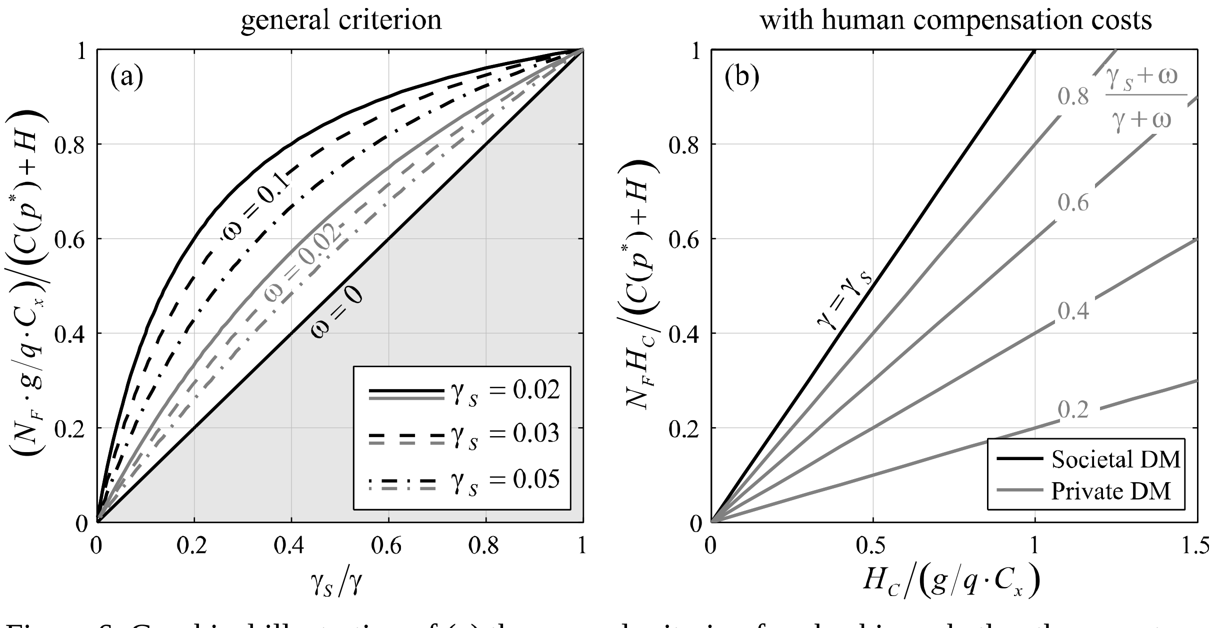

Figure 6: Graphical illustration of (a) the general criterion for checking whether the monetary optimum is acceptable from a societal point of view (equation (13)) and (b) when including human compensation costs in monetary optimization (equation (14)).

Figure 6 illustrates the behaviour of the rules derived above for checking whether the optimal decision is acceptable from a societal point of view. In Figure 6 (a), the threshold defined by equation (13) is investigated as a function of the ratio between the societal discount rate γ_S and the discount rate γ used for monetary optimization. The individual curves refer to different assumptions regarding the obsolescence rate ω and the societal discount rate γ_S. For many applications it may be sufficient to assume that $\gamma_S/\gamma = (\gamma_S + \omega)/(\gamma + \omega)$ and to check only whether the discount rate ratio γ_S/γ is larger than the ratio between the SWTP to prevent a loss of N_F lives and the total failure costs $C(p^*) + H$. The grey triangle contains all situations where, on the safe side and without making assumptions on the obsolescence rate ω, it can be assumed that the monetary optimum is an acceptable solution.

For a societal decision maker γ_S/γ equals one and criterion (13) simplifies to checking whether $g/q \cdot C_x \cdot N_F$ is larger than $C(p^*) + H$. This situation can be investigated further by introducing human compensation costs as a share of the total failure costs. In Figure 6 (b), the equality in criterion (14) is plotted as a function of the ratio between human compensation costs H_C and the SWTP per life saved. The black line refers to a societal decision-maker (i.e. $\gamma = \gamma_S$), the area below the line including all situations where a societal cost optimization leads to acceptable decisions following the LQI principle. The ratio $N_F H_C / (C(p^*) + H)$ on the y-axis cannot become larger than one. Thus for a societal decision-maker it is only necessary to impose the LQI criterion as a boundary condition to monetary optimization if the SWTP used in the acceptance criterion is larger than the human compensation costs H_C entering monetary optimization.

For a private decision-maker, one of the grey lines in Figure 6 (b) becomes relevant, as he will typically adopt a discount rate larger than the societal rate γ_S, thus $(\gamma_S + \omega)/(\gamma + \omega) < 1$. The higher the private discount rate γ, the more likely it is that the LQI criterion becomes active and that the decision-maker has to adopt a solution that is safer than the monetary optimum. Again the assumption $(\gamma_S + \omega)/(\gamma + \omega) = \gamma_S/\gamma$ may serve as an approximation, especially if the obsolescence rate ω is expected to be small.

4. Discussion and Conclusions

Based on the generic formulation introduced in section 2.2 and the numerical study presented in section 3.1, minimum target reliabilities can be defined based on the LQI. After introducing a possible format for LQI target reliabilities in different structural classes, the strength and limitations of the approach chosen will be shortly discussed.

4.1 Deriving target reliabilities from the LQI

A simple format for defining minimum target reliabilities from the LQI acceptance criterion can be derived by relating the LQI threshold reliability to the constant K_1 as defined in equation (10). Table 3 presents results for different "relative life saving costs classes".

Relative life saving costs	**Range for K_1 constant**	**LQI target reliability**
Large (I)	$10^{-3} \div 10^{-2}$	$\beta = 3.1\left(P_f \approx 10^{-3}\right)$
Medium (II)	$10^{-4} \div 10^{-3}$	$\beta = 3.7\left(P_f \approx 10^{-4}\right)$
Small (III)	$10^{-5} \div 10^{-4}$	$\beta = 4.2\left(P_f \approx 10^{-5}\right)$

Table 3: Tentative minimum target reliabilities related to one year reference period and ultimate limit states, based on the LQI acceptance criterion.

The cost classes are defined in terms of a range for the K_1 constant (second column of Table 3) and are valid for medium variabilities of the total loads and resistances (i.e. $0.1 \le V \le 0.3$). The target probability of failure may be increased by a factor 5 for higher coefficients of variation of the basic random variables. For low variabilities, on the other hand, it should be reduced by a factor 2. The values in Table 3 have been derived based on lognormally distributed loads and resistances; however they can serve also as a good approximation if other distribution types are used (see Figure 5 and discussion).

The approach followed in Table 3 is to define structural classes according to the relative life saving costs. Alternatively, due to the almost linear dependence between the constant K_1 and the LQI threshold for small coefficients of variation V_R and V_S, it is also possible to estimate the LQI target reliability based on the following approximate formula (again for $0.1 \le V \le 0.3$):

$$P_{f,acc} \approx \frac{K_1}{5} = \frac{1}{5} \cdot \frac{C_1\left(\gamma_S + \omega\right)}{g/q \cdot C_x \cdot N_F} \tag{15}$$

Independently of the approach followed, the LQI target reliability format should be accompanied by a simple rule for checking whether risk to life is extraordinarily high or, at the other extreme, negligibly small. Such a rule could be formulated in terms of the expected number of fatalities N_F and should be applied especially in situations where the constant K_1 is very small or large (e.g. below 10^{-5} or above 10^{-2}). If the human consequences in case of failure are very high, the simplified format introduced in this section should not be applied. Instead, the marginal life saving costs should be estimated based on direct risk assessment.

The LQI target reliabilities can be used either as a boundary condition for direct monetary optimization or in combination with the JCSS target reliabilities. The optimal design should be aimed at if the resulting reliability is higher than the LQI threshold. It has to be emphasized that the cost classes in Table 3 are not equivalent to those defined in the JCSS table: The constant K_1 is a measure relating the safety costs to the monetized human consequences in case of failure. The risk reduction costs in Table 1, on the other hand, are related to the total construction costs and the consequence assessment is used to choose one of the consequence classes. Simple criteria for checking whether the optimum is also acceptable following the LQI criterion have been discussed in section 3.2 (equation (13) and (14)).

4.2 Discussion and open questions

The biggest advantage of the proposed LQI target reliability format is its simplicity. The approach chosen for deriving the LQI threshold is highly generic and applicable to a broad range of decision problems where the yearly probability of "failure" (an adverse event leading to both human and monetary consequences) can be reduced at a cost. The generic formulation introduced in section 2 is based on the assumption that structures are systematically renewed after failure or obsolescence. The derivation of the LQI target reliabilities is further based on the assumption that the failure probability can be estimated using a simple $R - S$ limit state function. For the interaction between the LQI criterion and monetary optimization discussed in section 3.2, this assumption is not necessary. Therefore, the criteria derived for checking whether the LQI criterion is active can be applied independent of the method or limit state function used to assess the probability of failure.

The derivation of LQI target reliabilities as a function of the "relative life saving costs constant" K_1 allows for a quantitative definition of structural classes according to the socioeconomic capacity of a society for investments into life safety. For a specific structure, K_1 can be estimated as a function of the "relative costs of safety measure", see Table 2. The ratio C_1/C_0 has already been used for defining the cost classes in the JCSS target reliability table, which is based on monetary optimization. A first guess for C_1 can be made based on considerations regarding the increase in construction costs if the material used for the structure is doubled. Of course C_1 and K_1 can also be estimated based on a more elaborated physical modelling approach.

The LQI target reliability format proposed in section 4.1 can in principle be used both for failure at component level and for structural failure. The only requirement is that the safety costs C_1 and the human consequences given failure, N_F, are estimated for the same failure event. The cost estimation is easiest when looking at ultimate limit state failure at component level. Performing the consequence assessment at component level does, however, require taking into account structural robustness and a certain probability of escape, especially if the failure is ductile. For consistency with the JCSS target reliabilities, it is proposed to use the results presented in this paper for failure on structural level. As an approximation, they may also be applied to the failure of "critical" components dominating the failure mode on structural level.

Another open question is how to deal with the interaction between the LQI criterion and monetary optimization for societal decision-making. This question essentially boils down to the problem of how to determine human compensation costs H_C in the context of a societal cost benefit analysis, which is the subject of a second paper presented at the LQI Symposium as a discussion note [11]. Choosing a value for H_C equal to or larger than the SWTP makes the separate evaluation of the acceptance criterion superfluous. Using the LQI acceptance criterion as a boundary condition for monetary optimization (Figure 1) would of course still be relevant for private decision-makers.

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