The LQI acceptance criterion and human compensation costs for monetary optimization – A discussion note

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Abstract

When used for decision-making, the LQI acceptance criterion typically interacts with monetary optimization. At present, there is no clear consensus in the literature on how to combine these two parts of the decision problem. The goal of this note is to provide the basis for a structured discussion of this question. The outcome of the discussion at the Symposium is included in the conclusion.

1. Introduction

A question arising in almost all practical applications of the LQI is how to combine the societal acceptance criterion with monetary cost-benefit optimization. Despite its obvious importance, a clear answer to this question has not yet been given by the research community. Two fundamentally different approaches can be distinguished for dealing with the interaction between the two parts of a decision problem:

- The LQI criterion can be used within the framework of monetary optimization
- The LQI criterion can be used as a boundary condition for optimization

When following the first approach, the LQI criterion is used to transform the expected loss of lives or life years into monetary units. The monetized life loss then enters optimization simply as a cost (or benefit) term, see e.g. Nathwani et al [1]. The second approach, illustrated in Figure 1, uses the LQI criterion as a threshold defining the acceptable region within which monetary optimization is allowed, see Rackwitz [2] and others.

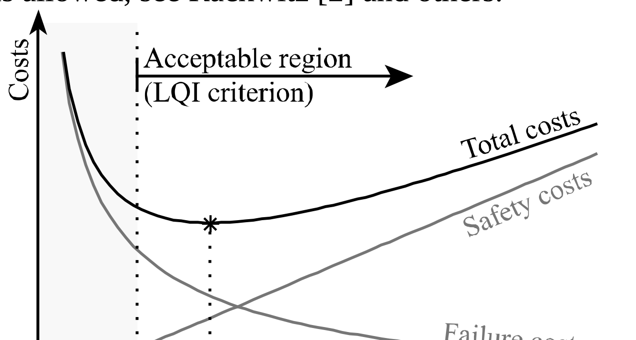

Figure 1: LQI criterion as a boundary condition for monetary optimization.

Risk acceptance always has to be evaluated from a societal point of view. Therefore, it makes sense to use the LQI criterion as a (societal) boundary condition especially if the optimization is performed by a private decision-maker. In addition to checking societal risk acceptance, the private decision-maker may include risk to life also in the optimization problem, e.g. if a compensation has to be paid to the relatives of the victims in case of a fatality.

The interaction between the acceptance criterion and monetary optimization is less clear for a societal decision-maker performing the optimization. The focus of the present paper is on this situation. As a starting point, we assume that the LQI acceptance criterion is applied as a boundary condition also for monetary optimization on a societal level, i.e. the second of the two approaches mentioned above is used. Monetized human consequences may of course still enter the objective function used for optimization. This allows a comparison with the first approach if societal human compensation costs are derived from the LQI.

The LQI threshold criterion requires that all efficient life saving investments have to be performed. What is efficient is judged by comparing the marginal life saving costs $-dg$ with the Societal Willingness To Pay (SWTP) derived from the LQI:

$$-dg \geq SWTP = \frac{g}{q}\frac{d\overline{e}_d}{\overline{e}_d} \tag{1}$$

Here, g denotes the gross domestic product per capita and $\overline{e}_d$ refers to the age-averaged discounted life expectancy. The LQI exponent defining the trade-off between wealth and longevity, q, is a function of the share of total life time spent for work, w, and the output elasticity of labour, β, in a Cobb-Douglas production function. The corresponding LQI definition is the following:

$$L = g^q \overline{e}_d \qquad q = \frac{1}{\beta}\frac{w}{1-w} \tag{2}$$

As a basis for the discussion we assume the LQI acceptance criterion and the formulation of the LQI to be given. In the following, we concentrate on the monetary optimization part of the decision problem and on its interaction with the acceptance criterion. We start with a review and discussion of different approaches for quantifying human compensation costs based on the LQI. After discussing the implications of different compensation cost values for the interaction between risk acceptance and monetary optimization, we come back to the question whether the LQI criterion should be used as a boundary condition for cost optimization also in the case of a societal decision-maker performing the optimization.

2. Societal human compensation costs in monetary optimization

The definition of compensation costs for private decision makers is clear: It is the amount of money that in case of a fatality has to be paid by the decision-maker, e.g. to the relatives of the victim. From a societal point of view, such compensation costs are transfer payments that cancel out when aggregating costs and benefits over all members of a society. It is thus not possible to simply apply the same definition of compensation costs for the case of a societal decision-maker performing the optimization. Nevertheless, also at societal level it seems reasonable to assume a nonzero monetary value to account for the loss of human lives in monetary optimization, even if the value is defined based on purely economic considerations, i.e. regarding humans as production factors. In the following, the term "(societal) human compensation costs" will be used to make clear that the value enters monetary optimization (and not the LQI acceptance criterion), even though compensation is clearly not a meaningful concept in the context of societal decision-making.

Several approaches for estimating human compensation costs have been proposed in the LQI literature. The individual authors not only use different terminology, also the quantitative values proposed vary considerably. In the following the different approaches will be reviewed and discussed both with respect to their consistency with the LQI formulation (2) and to their implication for the interaction between the acceptance criterion and monetary optimization.

2.1 Review of approaches discussed in the LQI literature

Table 1 gives an overview over different approaches proposed in the LQI literature for estimating societal human compensation costs. The references cited are selected to provide information both on the derivation of the formulas in Table 1 and their application in the context of monetary optimization. The quantitative values are derived using statistical information for Switzerland. The respective input values are $g = 69'887 CHF$, $w = 0.12$, $\beta = 0.72$ and $q = 0.19$. Swiss period life tables were used for demographic calculations, with a discount rate of 5%, where required (intra-generational discounting).

References	Formula	Value [Mio. CHF]	Value / SWTP
Nathwani et al. [1], Rackwitz [3]	$SWTP_{1\text{life}} \approx \frac{g}{q} C_\Delta$	4.98	1.00
Faber [4], Rackwitz [2]	$SVSL = \frac{g}{q} \bar{e}_d$	5.52	1.11
Lentz [5], Rackwitz [2]	$SHC = g\bar{e}$	2.89	0.58
Skjong and Ronold [6]	$ICAF = g \frac{1-w}{w} \frac{1}{2} \cdot \frac{e_0}{2}$	10.19	2.05
Ditlevsen [7]	$ICAF = g \frac{1-c}{c} \frac{1+V_{e_0}^2}{2} e_0$	6.77	4.26
Rackwitz [2]	$SLSC \approx \Delta e \cdot g \left[1-\left(1+\Delta e/e_0\right)^{-1/q}\right]\Big\vert_{\Delta e = e_0/2}$	2.46	0.49
Kübler [8]	$SLSC^{(-)} \approx -\Delta e g \left[1-\left(1-\Delta e/e_0\right)^{-1/q}\right]\Big\vert_{\Delta e = e_0/2}$	103.34	20.77

Table 1: Different approaches to derive human compensation costs from the LQI.

In the last column of Table 1, the different values for the human compensation costs are related to the SWTP (Societal Willingness to Pay) to save one additional life that can be derived from the acceptance criterion (1). Here, the ratio $d\bar{e}_d / \bar{e}_d$ is estimated using the demographic constant (C_Δ) approximation by Rackwitz [3]. Nathwani et al. [1] introduce the concept of SCCR (Societal Capacity to Commit Resources), which is closely related to the SWTP, for monetizing human losses in the context of monetary optimization. Differences in notation and the estimation of $d\bar{e}_d / \bar{e}_d$ are neglected in our summary.

The SVSL (Societal Value of a Statistical Life) is derived from the same LQI formulation as the SWTP, but with a slightly different interpretation: While the LQI is treated as a bivariate utility function in the derivation of the SWTP, in the SVSL concept only the monetary part, $u(g) = g^q$, is seen as a utility function. The multiplication with life expectancy is understood to be an intertemporal extension of the one-period model. The SVSL can be derived by dividing the “life time utility”, $g^q\bar{e}_d$, by the marginal utility of wealth, $\partial u/\partial g$ in order to transform it to monetary units. Rackwitz [2] concludes that the SWTP and the SVSL are based on fairly similar concepts and therefore complement each other. Faber [4] proposes to use the SVSL for deriving human compensation costs from the LQI.

Lentz [5] proposes the SHC (Societal Human Capital) as a lower bound for compensation costs. Loss of life is here evaluated in terms of lost future earnings due to the premature death of a person. The SHC is thus not directly related to the LQI but may nonetheless be regarded to be consistent with the derivation of the LQI, see Rackwitz [2]. Following Lentz [5], future life years are not discounted in the SHC formula in Table 1 because income is expected to grow with the same (or at least a similar) rate as the one used for discounting.

The ICAF (Implied Costs of Averting a Fatality) formulation by Skjong and Ronold [6] is based on an earlier definition of the LQI, which explains why future life years are not discounted and the ratio $w/(1-w)$ is used instead of q. Age-averaging is performed implicitly by approximating de with $e/2$. The final multiplication with $de \approx e/2$ is justified by Skjong and Ronold’s interpretation of the LQI criterion as giving acceptable costs per life year saved. The ICAF formulation by Ditlevsen [7] uses a more elaborate approach for estimating de/e based on a random variable representation of the time to a fatal accident and the time to death without the accident. For the calculations in Table 1 it was assumed that $V_{e_0} = 0.2$ and $c = 0.3$ in consistency with the values proposed by Ditlevsen [7].

Rackwitz’ SLSC (Societal Life Saving Costs) are derived from the equality in (1) by separating and integrating from g to $g + \Delta g$ and from e to $e + \Delta e$. The resulting value is multiplied with Δe based on similar considerations as behind Skjong and Ronold’s ICAF. Age-averaging can be performed explicitly or approximately by setting Δe to $e/2$, see Rackwitz [2]. As discussed by Kübler [8], the SLSC formulation implies that the change in life expectancy is not marginal. Inserting a negative Δe leads to a different value (termed $\text{SLSC}^{(-)}$ in Table 1) because the LQI indifference curves are nonlinear. While Rackwitz’ SLSC gives the Willingness To Pay for increasing life expectancy, Kübler’s modified SLSC can be interpreted as the Willingness To Accept a non-marginal decrease in life expectancy.

2.2 Consistency with the acceptance criterion

Of the approaches for quantifying human compensation costs from the LQI summarized in Table 1, the SWTP and the SVSL concept are clearly consistent with the LQI formulation in equation (2). With respect to the SVSL it may however be questioned why the LQI should be interpreted differently in the two parts of the decision problem. Treating the LQI as a bivariate utility function, as in the SWTP derivation, seems to be more consistent with the work-leisure time optimization principle used to derive the LQI exponent q. The SHC concept can be regarded to be consistent with the acceptance criterion, too, although it might be argued that discounting should be performed for consistency with the LQI formulation (2).

An assumption common to the remaining approaches in Table 1 is that the LQI criterion is regarded to give annual values for the SWTP. With respect to life saving benefits achieved in future years this is true: The acceptance criterion (1) determines the fraction $-dg$ of the (yearly) GDP that society is willing to invest in order to achieve an increase in life expectancy $d\bar{e}_d$ that is implied by a reduction $-d\mu$ in (yearly) mortality. Thus, the yearly SWTP values have to be aggregated over the service life of the safety measure if it shall be compared e.g. with an initial investment. The fact that saving a life is equivalent to saving $\bar{e}_d$ (discounted) future life years, on the other hand, is already accounted for in the acceptance criterion (or, more precisely, the demographic constant C_Δ). Therefore, the ICAF and SLSC values in

Table 1 have to be divided by a measure for life expectancy (e_0 or $e_0/2$, depending on the approach) for intertemporal consistency with the LQI acceptance criterion.

Introducing discounting into the four ICAF and SLSC formulas is another step that has to be performed to achieve consistency with the LQI formulation (2). When regarding Skjong and Ronold's ICAF, this leads to an expression equivalent either to the SWTP or to the SVSL formula. Discounting future life years in the random variable representation proposed by Ditlevsen is less straightforward. Thus, adapting Ditlevsen's ICAF to an LQI definition based on age-averaged discounted life expectancy seems to be not a trivial task.

The SLSC formulas proposed by Rackwitz and Kübler can be theoretically adapted to account for discounting and age-averaging. The assumption of a non-marginal change in life expectancy, however, can be seriously questioned in most practical applications (maybe excluding Kübler's asteroid example [8]). From the point of view of an individual facing death, the change in life expectancy is of course large. However, from a societal point of view the loss of one life (or even a fairly large number of lives) is clearly marginal when compared to background mortality. The difference between Willingness To Pay (SLSC) and Willingness To Accept (SLSC $^{(-)}$) disappears if the life expectancy change is assumed to be marginal.

2.3 Implications for the interaction with the LQI criterion

Qualitatively, the implications of choosing a high or low value for the compensation costs are clear: The less risk to life is accounted for in monetary optimization, the more likely it is that the LQI acceptance criterion becomes active. In the following, a simple quantitative criterion for checking whether the optimal solution is also acceptable is derived as a function of the SWTP defined by the LQI criterion and the societal human compensation costs value H_C used in monetary optimization. The focus is still on the situation of a societal decision-maker performing the optimization. Therefore, the effect of discounting can be neglected, because the discount rate is the same for the acceptance criterion and monetary optimization.

Following the "total cost" representation in Figure 1, the objective function for monetary optimization can be formulated as follows:

$$\min_p \left\{ C(p) + R(p) \right\} = \min_p \left\{ C(p) + \left(R_M(p) + R_H(p) \cdot H_C \right) \right\} \tag{3}$$

Here, $C(p)$ are the safety investments, $R(p)$ denotes the failure costs or (monetary) risk and p is the decision parameter. For simplicity, the monetary benefit is neglected (it may be assumed to be either independent of p or included in $R(p)$). It is furthermore assumed that all components of the objective function have already been aggregated and discounted to the decision point using a societal discount rate. The risk $R(p)$ can be split into a monetary part $R_M(p)$ and the risk to human life $R_H(p)$. Life risk is monetized by multiplying with the human compensation costs per fatality, H_C. The optimal decision fulfils the following condition:

$$\frac{dC}{dp} + \frac{dR_M}{dp} + \frac{dR_H}{dp} \cdot H_C = 0 \tag{4}$$

Dividing by the total population size N_P gives average per capita values (the denominator dp is omitted for ease of notation):

$$\frac{dC}{N_P} + \frac{dR_M}{N_P} + d\mu \cdot H_C = 0 \tag{5}$$

As discussed in Fischer et al. [9], the marginal life saving costs dg in the LQI criterion (1) have to be derived from the safety investments only ($-dg = dC/N_P$), i.e. excluding marginal reductions in monetary risk. With this definition it is now possible to insert (5) into (1), which leads to the following criterion for checking whether the optimal solution is acceptable according to the LQI criterion:

$$-\frac{dR_M}{N_P} - d\mu \cdot H_C \geq -d\mu \cdot SWTP_{1\text{life}} \tag{6}$$

The LQI criterion is most likely to become active if there is no monetary benefit of increasing safety besides the monetized human consequences. Setting dR_M to zero and dividing by $-d\mu > 0$ leads to the following result:

$$H_C \geq SWTP_{1\text{life}} \tag{7}$$

This implies that the LQI criterion will never be active if the compensation costs H_C are chosen to be equal to or larger than the SWTP per life saved used in the LQI criterion.

3. Conclusion

As a starting point for this discussion note we assumed that the LQI acceptance criterion for risk to life is used as a boundary condition for monetary optimization. This approach clearly makes sense for a private decision-maker performing the optimization: In a market economy monetary optimization is generally left to the individual within certain bounds defined by societal risk acceptance criteria. The situation is less clear for a societal decision-maker performing the optimization, which is the focus of the discussion in the present paper.

At a first glance, for societal decision-makers it seems to be more intuitive to directly include the LQI criterion in monetary optimization. The advantage of using the LQI to define a boundary condition for optimization is that the same acceptance criterion can be used for all decision-makers. For a consistent allocation of resources for life safety in different sectors of society, it is thus clearly beneficial to separate risk acceptance from monetary optimization also for societal decision-makers. For all decisions made in society, risk acceptance is then guaranteed by imposing the LQI criterion as a boundary condition for optimization. Independent of this conclusion, the potential loss of human lives may of course also be included in the optimization. Different approaches for monetizing human consequences in the context of optimization from a societal point of view have been reviewed and discussed in this paper. Finding a clear definition for these “societal human compensation costs” is not an easy task, because the notion of human compensation itself is not a meaningful concept in the context of societal decision-making. In the discussion at the Symposium it was concluded that the LQI should primarily be applied to define the acceptance criterion as a societal boundary condition and not to define human compensation costs for monetary optimization.

Nevertheless it may be helpful both for societal and private decision-makers to use a monetary value for including the potential loss of human lives in optimization that can be regarded to be consistent with the LQI acceptance criterion. Two possible approaches for quantifying human compensation costs in consistency with the LQI are the Societal Human Capital (SHC) approach and the Societal Willingness To Pay (SWTP) that is also used in the acceptance criterion. Choosing the first value can be justified if the possible loss of human lives shall be evaluated only from a financial perspective, i.e. regarding humans as production factors. Societal risk acceptance then has to be guaranteed by imposing the LQI criterion as a boundary condition. Of course the LQI acceptance criterion also holds when using the SWTP approach to include human consequences in optimization. As could be shown in this paper, the optimal decision will then however always be acceptable according to the LQI criterion, at least if all costs and benefits are evaluated and discounted in the same way in both parts of the decision problem. In the case of a societal decision-maker performing the optimization, this condition is clearly fulfilled.

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