Risk optimization of road tunnels using LQI

MILAN HOLICKÝ

Klokner Institute, Czech Technical University in Prague
Šolínova 7, 166 08 Prague 6, Czech Republic
Milan.Holicky@klok.cvut.cz

Abstract
Probabilistic methods of risk optimization are applied to identify the most effective safety measures considered in the design of road tunnels. A general procedure is illustrated by the optimization of a number of escape routes using the concept of Life Quality Index. It appears that the discount rate and specified life time of a tunnel affect the total consequences and the optimum arrangements of the tunnel more significantly than the number of escape routes.

1. Introduction

Tunnel structures usually represent complex technical systems that may be exposed to hazard situations leading to unfavourable events with serious consequences. Minimum safety requirements for tunnels in the trans-European road network are provided in the Directive of the European Parliament and of the Council 2004/54/ES [1]. The Directive also gives recommendations concerning risk management, risk assessment and analysis.

Methods of risk assessment and analysis are more and more frequently applied in various technical systems ([2], [3]) including road tunnels ([4], [5], [6], [7]). The submitted contribution attempts to apply methods of probabilistic risk optimization using the results of previous studies ([6], [9] and [10]) of risk assessment. An essential extension of the presented study consists of the application of the concept of Life Quality Index (*LQI*) based on the recent studies ([8], [9], [10] and [11]).

2. Principles of risk optimization

The total consequences $C_{tot}(k,p,n)$ relevant to the construction and performance of a tunnel are generally expressed as a function of the decision parameter k defined as the number of escape routes, the annual discount rate p (commonly about $p \approx 0{,}03/\text{a}$) and the life time n (commonly $n = 100$ years).

The fundamental model of the total consequences is written as a sum of partial consequences

$$C_{tot}(k,p,n) = R(k,p,n) + C_0 + \Delta C(k) \tag{1}$$

In equation (1) $R(k,p,n)$ denotes the expected societal risk, which is dependent on the parameter k, discount rate p and life time n. C_0 denotes the initial construction cost independent of k, and $\Delta C(k)$ additional expenses dependent only on k (not on the discount rate p and life time n as these are expenses spent during construction).

The societal risk $R(k,p,n)$ may be estimated using the following formulae

$$R(k,p,n) = N(k)\, R_1\, Q(p,n), \quad Q(p,n) = \frac{1 - 1/(1+p)^n}{1 - 1/(1+p)} \tag{2}$$

In equation (2) $N(k)$ denotes the number of expected fatalities per one year (dependent on k), R_1 denotes the so called Societal Value of Statistical Life (*SVSL*) – an acceptable compensation cost for one fatality, and p the discount rate (commonly within the interval from

0 to 5 %). The quotient q of the geometric row is given by the fraction $q = 1/(1+p)$. The discount coefficient $Q(p,n)$ makes possible to express the actual expenses during a considered life time n in current cost considered in (1). Thus, expenses R_1 in a year i correspond to the current cost $R_1\, q^i$. The sum of the expenses during n years is given by the coefficient $Q(p,n)$.

The key function $N(k)$ is dependent on a number of assumptions concerning consequences of adverse events and it may change when different hypotheses are accepted. In particular, some specific premises accepted for assessing the number of endangered persons and their ability to escape in case of fire (see studies [4], [5] and [6]) are essential.

A necessary condition for the minimum of the total consequences (5) may be given by the vanishing of the first derivative with respect to k written as

$$\frac{\partial N(k)}{\partial k} R_1 Q(p,n) = -\frac{\partial \Delta C(k)}{\partial k} \tag{3}$$

In some cases this condition may not lead to a practical solution, in particular when the discount rate p is small (the corresponding discount coefficient $Q(p,n)$ is large) and there is a limited number of escape routes k, which cannot be arbitrarily increased.

3. Standardized consequences

The total consequences given by equation (1) may be, in some cases, simplified to a dimensionless standardized form and the whole procedure of optimization may be generalized. Consider as an example the optimization of the number k of escape routes. It is assumed that the involved additional costs $\Delta C(k)$ due to k may be expressed as the product $k\, C_1$, where C_1 denotes the cost of one escape route. The cost C_1 is approximately equal to R_1 (assumed also in [6]). However, in the following both variables are considered as generally independent quantities. Generally it may be considered that equation (1) becomes

$$C_{\text{tot}}(k,p,n) = N(k)\, R_1\, Q(p,n) + C_0 + k\, C_1 \tag{4}$$

This function can be standardized as follows

$$\kappa(k,p,n) = \frac{C_{tot}(k,p,n) - C_0}{R_1} = N(k)Q(p,n) + k\zeta, \quad \zeta = \frac{C_1}{R_1} \tag{5}$$

Both variables $C_{\text{tot}}(k,p,n)$ and $\kappa(k,p,n)$ are mutually dependent and have the minimum (if it exists) for the same number of escape routes k. A necessary condition follows from (3) as

$$\frac{\partial N(k)}{\partial k} = -\frac{\zeta}{Q(p,n)} = -\zeta\, \frac{1 - 1/(1+p)}{1 - 1/(1+p)^n} \tag{6}$$

An advantage of the standardized consequences is the fact that it is independent of C_0. In addition to the number of escape routes k, discount rate p and assumed life time n, the standardized consequences $\kappa(k,p,n)$ are also dependent on the cost ratio $\zeta = C_1/R_1$, i.e. the ratio of the cost of one escape route C_1 and the Societal Value of Statistical Life $SVSL \approx R_1$, which can be based on the concept of Life Quality index, as indicated below.

4. Life quality index *LQI*

4.1. Principal form

Based on the theory of socio-economics, the Life Quality Index (*LQI*) can be expressed in the following principal form ([8] and [9]):

$$L(g,l) = g^{\,q}\, l \tag{7}$$

where g denotes the part of GDP per capita, which is available for risk reduction purposes, l is the expected life time and the parameter q is a measure of the trade-off between the resources available for consumption and the value of the time of healthy life. The parameter q may be assessed as:

$$q = \frac{1}{\beta} \frac{w}{1-w} \tag{8}$$

where β is a constant (around 0,7) taking into account the fact that only a part of the GDP is based on human labour (the other part is due to investments), w denotes a fraction of the life time devoted to work (around 0,15). Considering the above mentioned numerical values, the parameter q is about 0,25.

4.2. Societal Willingness To Pay *SWTP*

Assuming that any investment into life risk reduction should not lead to a decrease of LQI, the following risk acceptance criteria ([19]) may be obtained:

$$\mathrm{d}g + \frac{g}{q} \frac{\mathrm{d}l}{l} \geq 0 \tag{9}$$

Here notation dg may be also interpreted as a finite increment of g (similarly for all other variables). Using this relationship, the societal willingness to invest annually into life saving activities (Societal Willingness To Pay) may be assessed as [19]:

$$SWTP = \mathrm{d}g = -\frac{g}{q} \frac{\mathrm{d}l}{l} \tag{10}$$

The annual investment dg indicated by equation (10) denotes the annual cost the society is willing to pay (a negative expenditures for a society) to increase the life expectancy by dl.

The relative change dl/l may be replaced by a decrease in mortality dμ as [20]

$$\frac{\mathrm{d}l}{l} \approx D\, \mathrm{d}\mu = D\, \pi\, \mathrm{d}m \tag{11}$$

where dm is the rate of adverse events, D is a demographic economic constant for mortality reduction, and π denotes the probability of dying given an adverse event. Detail comment on equation (11) provided in [11] indicates that the constant D may be set approximately to 20. It follows from equations (10) and (11) that the annual investment into life safety denoted as dC (instead of $SWTP$) may be written as

$$\mathrm{d}C = -\frac{g}{q} D\, \mathrm{d}\mu = -\frac{g}{q} D\, N_{PE}\ p\, \mathrm{d}m \tag{12}$$

where N_{PE} denotes the number of persons exposed to the adverse event. In case of road tunnels the change in mortality dμ was determined directly using methods of risk assessment. If the failure rate depends on a decision parameter k, then the acceptance criterion is

$$\mathrm{d}C(k) \geq -\frac{g}{q} D\, \mathrm{d}N(k) \tag{13}$$

Here dC(k) denotes the amount that should be annually invested in life safety as a function of the number of escape routes k.

As an example, consider g, the part of GDP per capita, which is available for risk reduction purposes, be the notional value g = 10 000 USD. Assuming further the above mentioned parameter, and q = 0,25, and D = 20, then the annual investment

$$\mathrm{d}C(k) \geq -800\,000\, \mathrm{d}N(k) \tag{14}$$

Note that the criterion (13) or (14) serves as the limitation of a possible investment decision and not as a condition for compensation in case of accident.

The minimum annual investment d$C(k)$ decreases with the number of escape routes k from almost 80 000 USD to about 15 000 USD for 30 escape routes. It should be mentioned that the function $N(k)$ was obtained numerically and then approximated by a polynomial.

It should be noted that N(k) is the number of fatalities per year. Consequently, d$C(k)$ are yearly payments. But all the costs must be raised at a certain decision point. Then, privately financed and publicly financed projects should be distinguished. Private financing makes the tunnel more expensive due to the annuity and this aspect could be included in equation (8), but possibly with another discount rate (market rate).

It is interesting to note that equation (13) may be used to specify an acceptable number of escape routes k_{acc}. Equation (13) may be written in the form

$$C_1 \geq -\frac{g}{q} D \,\mathrm{d}N(k) \tag{15}$$

Equation (15) can be used to specify the number of escape routes k complying with the acceptance criterion (13). The minimum k satisfying equation (15), called the acceptable number of escape routes k_{acc}, should be considered as a lower bound for any specified number k of escape routes. The tunnel is acceptable if $k > k_{acc}$. This condition should be verified also for an optimum number of escape routes k_{opt} (derived below). Thus, the tunnel is acceptable if $k_{opt} > k_{acc}$, when $k_{opt} < k_{acc,}$ then k_{acc} should be accepted.

4.3. Societal value of Statistical Life *SVSL*

The *LQI* principles may be also used for the assessment of compensation costs ([11]). Considering the principal form of *LQI* given by equation (7), the Societal Value of a Statistical Life (*SVSL*) may be assessed as

$$SVSL = \frac{g}{q} E \tag{16}$$

In equation (16) E denotes the so-called age-averaged discounted life expectancy ([11]). In this concept, discounting means a societal discounting, roughly given by the natural discount rate. The age-averaging takes account of the fact that the age distribution of fatalities should mirror the age distribution of the population. Considering an effective discounting of 3% per annum, the discounted life expectancy is about 30 years and the corresponding *SVSL* is

$$SVSL = \frac{g}{q} E = 40\,000 \times 30 = 1\,200\,000 \text{ USD} \tag{17}$$

The *SVSL* is considered as an estimation of the compensation cost for one fatality R_1.
However, in an optimization concept R_1= *SVSL* may give a rather high value. The real compensation cost (carried by the social system of the state or an insurance) hardly exceeds the lost income in an event, approximately $g\ l/2$ =10000*70/2=350000 USD.

5. Risk optimization

The total consequences $\kappa(k,p,n)$ are indicated in Figure 1, showing the variation of the standardized total consequences $\kappa(k,p,n)$ given by equation (5) with the number of escape routes k for selected discount rates p (up to 5 %), and for the cost ratio $\zeta = C_1/R_1 \approx 1$.

It follows from Figure 1 that the minimum total consequences depend considerably on the discount rate p. For example, for the expected life time n = 100 years and the discount rate p = 0,05 the optimum number of escape routes k is about 14, for the discount rate 0,01 the optimum number of escape routes k is more than 40 (most likely an unrealistic solution as the corresponding distance of escape routes would be less than 100 m). However, if the expected life time, considered in the optimization, is 50 years only, then for the discount rate p = 0,01 the optimum number of escape routes k is about 30.

It appears that the cost ratio $\zeta = C_1/R_1$ of the cost of one escape route C_1 and the Societal Value of Statistical Life *SVSL* ≈ R_1 may affect the total consequences and the optimum number of escape routes even more dramatically than the discount rate and the expected life time [7].

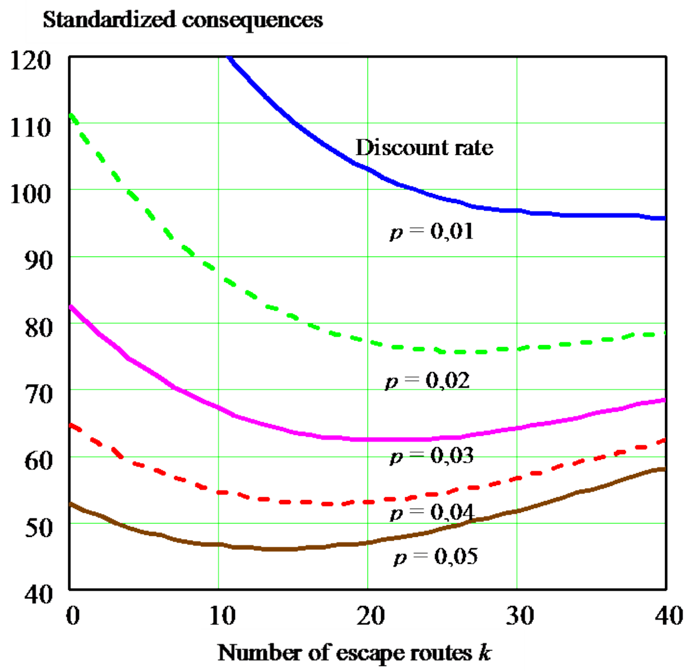

Figure 1: Variation of the standardized total consequences $\kappa(k,p,n)$ with the number of escape routes k for the cost ratio ζ = 1, selected discount rates p and the life time n = 100 years.

The optimum number of escape routes k increases with decreasing the cost ratio ζ (i.e. with decreasing the cost of one escape route C_1 or increasing the Societal Value of Statistical Life *SVSL* ≈ R_1). For example, for ζ = 2 (C_1= 2R_1) the optimum k is about 9, for ζ = 1 (C_1= R_1) the optimum k is about 20 and for ζ = 0,5 (C_1= 0,5 R_1) the optimum k is more than 40 (most likely an unrealistic solution as the corresponding distance of escape routes would be less than 100 m). Obviously, the correct specification of the cost of one escape route C_1 and the Societal Value of Statistical Life *SVSL* ≈ R_1 is of the uttermost importance.

6. Conclusions

The concept of Life Quality Index *LQI* and derived notions of the Societal Willingness To Pay SWTP and Societal Value of Statistical Life *SVSL* seem to provide an effective and powerful tool to balance societal and economic aspects. It appears that the cost ratio of the cost of one escape route and *SVSL*, the assumed life time and the discount rate may significantly affect the total consequences and the optimum number of escape routes.

However, as tunnels are commonly designed for a long period of time (100 years), probabilistic risk optimization may be applicable only if relevant life time and discount rate are available. The following conclusions should be, therefore, considered as qualitative (informative) statements only.

- The requirement for the minimum total consequences provides valuable information for the specification of the optimum number of escape routes.
- The total consequences are primarily affected by the cost ratio between the cost for one escape route and acceptable societal compensation cost *SVSL*, and the discount rate, less significantly by the assumed life time and the number of escape routes.
- The optimum number of escape routes depends primarily on the ratio between the cost for one escape route and acceptable societal compensation cost *SVSL*, partly on the discount rate and assumed life time.
- The compensation cost $R_1 = SVSL$ may give a rather high value. The real compensation cost (carried by the state or insurance) hardly exceeds the lost income in an event.

A correct specification of the discount rate and required life time is essential for making proper decisions. Further investigations of models for their societal and economic consequences are needed. In particular, consistent values for the costs of various safety measures and compensation costs *SVSL* based on Life Quality Index are required.

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References

[1] Directive 2004/54/EC of the European Parliament and of the Council of 29 April 2004 on minimum safety requirements for tunnels in the trans-European road network. Official Journal of the European Union L 201/56 of 7 June 2004.
[2] Stewart M.S. & Melchers R.E.. Probabilistic risk assessment of engineering system, Chapman & Hall, London, 1997, pp 274.
[3] Melchers R.E. 1999. Structural reliability analysis and prediction. John Wiley&Sons, Chichester, 1999, pp 437.
[4] Kruiskamp M.M., Weger D. de & Hoeksma J. TUNprim: A Spreadsheet model for the calculating road tunnels risk, Tunnels Management International, Volume 5, Number 1, 2002, pp. 6 – 12.
[5] Vrouwenvelder A.C.W.M. & Krom A.H.M. 'Hazard and the Consequences for Tunnels Structures and Human Life', 1st International Symposium Safe and Reliable Tunnels in Prague, CUR, Gouda, The Netherlands, 2004.
[6] Holický M. & Šajtar L. 2006 Probabilistic Risk Assessment and Optimization of Road Tunnels', Advances in Safety and Reliability, ESREL 2006, Taylor & Francis Group, London, 2006, pp. 2065-2071.
[7] Holický, M. Probabilistic risk optimization of road tunnels, Structural Safety 31 (2009) 260–266
[8] Rackwitz, R. Optimization and Risk Acceptability Based on the Life Quality Index. Structural Safety, 2002, Vol. 24, pp. 297-332.
[9] Rackwitz, R. The Philosophy Behind the Life Quality Index and Empirical Verifications. Memorandum to JCSS. Munich, Germany, 2005.
[10] Rackwitz, R., Lentz, A. and Faber, M.H. Socio-economically sustainable civil engineering infrastructures by optimization. Structural Safety, 27(3), July 2005, pp. 187-229.
[11] Rackwitz, R., Faber, M.H. and Schubert, M. Prinzipien der Sicherheitsbewertung. Bundesamt für Strassenwesen ASTRA. January 2007.