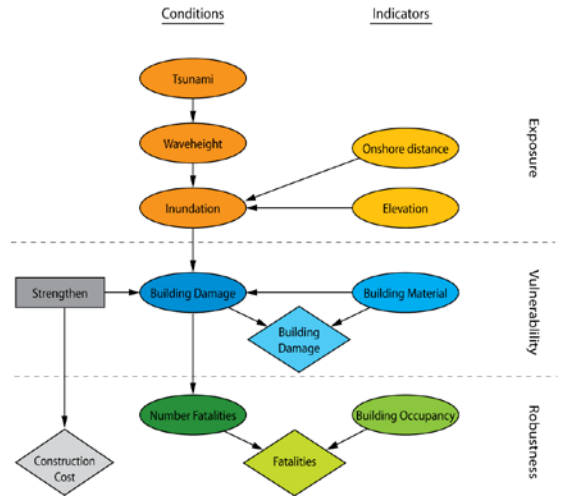


The LQI Symposium
DTU, August 21-23, 2012

In Memory of Professor Rüdiger Rackwitz

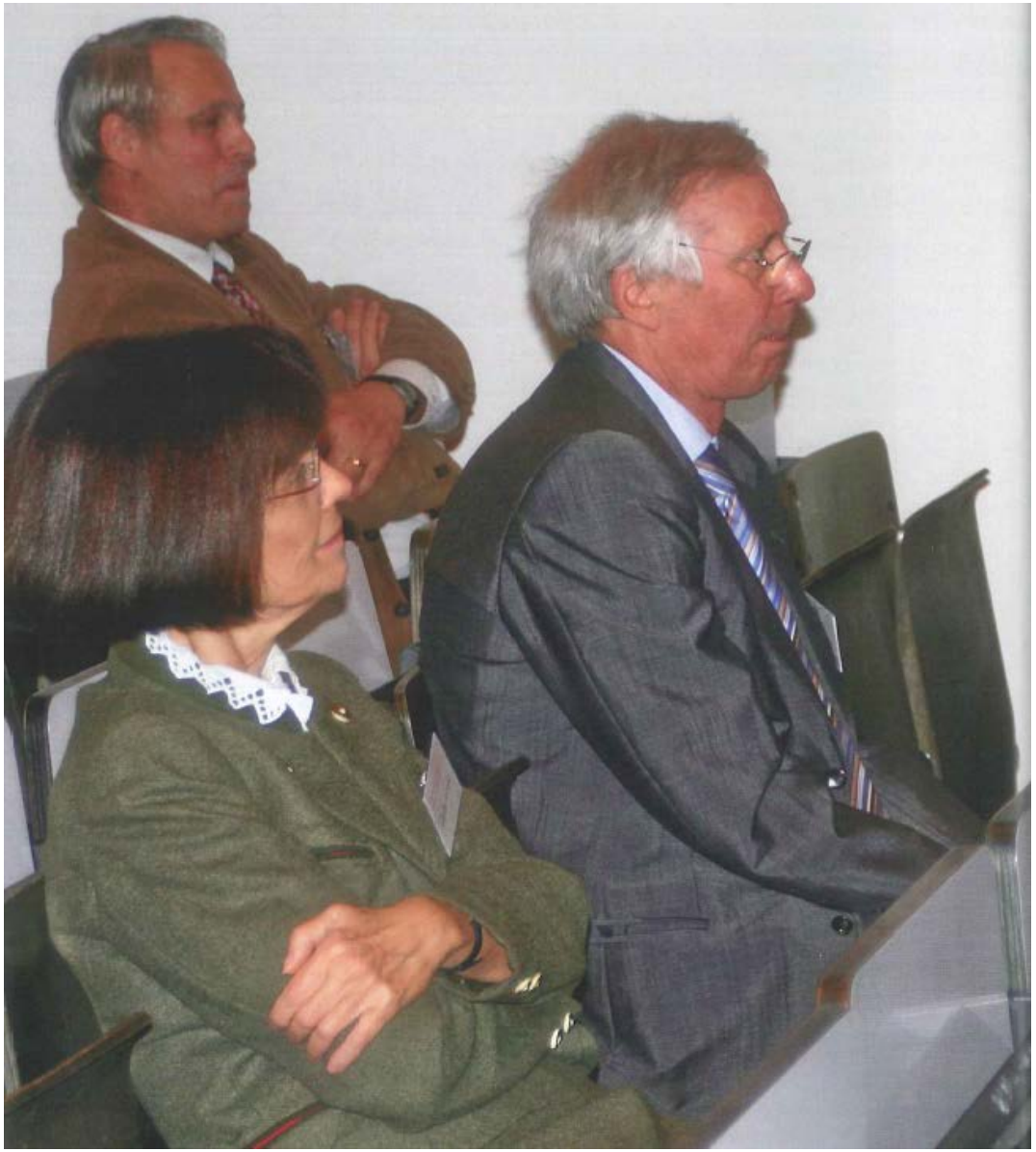
DTU

Program

Tuesday, August 21	
13:30-14:00	Opening
14:00:14:30	Origin and Development of the LQI, **Niels Lind, Jatin Nathwani**
14:30-14:45	Discussion
14:45-15:15	The LQTAI Principle **Michael H. Faber**
15:15-15:30	Discussion
15:30-16:00	Coffee break
16:00-16:30	On The Observed Stochastic Balance Between Work And Free Time **Ton Vrouwenvelder**
16:30-16:45	Discussion
16:45-17:15	Discount Rates For Use In Calculating the J-Value **Philip Thomas**
17:15-17:30	Discussion

Wednesday, August 22	
09:00-09:30	The LQI Acceptance Criterion And Human Compensation Costs For Monetary Optimization **Katharina Fischer and Michael H. Faber**
09:30-09:45	Discussion
09:45-10:15	Bio-Safery In Animal Labs **Tareq Aljazar**
10:15-10:30	Discussion
10:30-11:15	Coffee break
11:15-11:45	Risk Optimization Of Road Tunnels Using LQI **Milan Holicky**
11:45-12:00	Discussion
12:00-13:15	Lunch
13:15-13:45	Experience With Implementing Use Of LQI At The IMO **Rolf Skjong**
13:45-14:00	Discussion
14:00-14:30	Overview Of Literature On The LQI And Rational Risk Regulation - how to proceed **Niels Lind**
14:30-14:45	Discussion
14:45-15:15	LQI - The marginal Life Saving Cost Principle In Engineering Practice **Matthias Schubert**
15:15-15:30	Discussion
15:30-16:00	Coffee break
16:00-16:30	Deriving Target Reliabilities From The LQI **Katharina Fischer, Celeste Bernardo and Michael H. Faber**
16:30-16:45	Discussion
16:45-17:15	On The Compatibility Of The LQI With Present Best Practice Legislation **Michael H. Faber**
17:15-17:30	Discussion

Thursday, August 23	
09:00-09:30	Life Health Management In A Global Community **Michael H. Faber**
09:30-09:45	Discussion
09:45-10:30	Conclusions - main difficulties in advancing the use of the LQI
10:30-11:15	Coffee break
11:15-11:45	Conclusions - need for further developments
11:45-12:00	Closure



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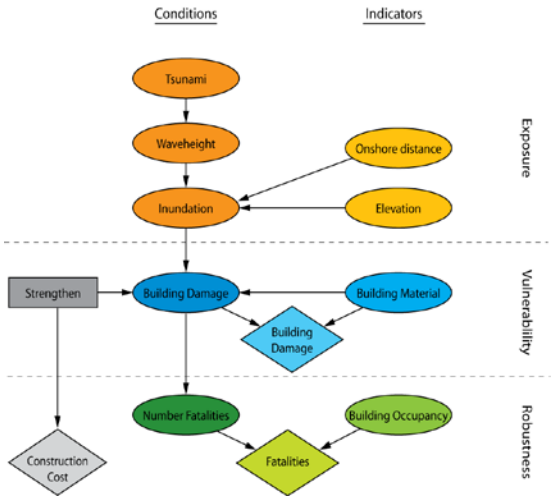


The LQTAI Principle

presented for Ove Ditlevsen based on

**Ove Ditlevsen and Peter Friis-Hansen:
Cost and benefit including value of life, health and environmental damage measured in time units.
Structural Safety 31 (2009) 136-142**

M. H. Faber

**Professor of Risk and Safety
Head of Civil Engineering
DTU, Denmark**

Contents of Presentation

- Introduction and basic idea
- The work time to free time in good health equilibrium
- The invariance principle
- Conclusions

Introduction and Basic Idea

- **The Life Quality Time Allocation Index (LQTAI) may be understood as a generalization of the LQI**

- **The societal preference for investing societal resources into life saving activities is expressed not in monetary terms but rather – by normalization – in time units**

- **The concept of "worth" rather than value is moreover introduced to account for the relative benefit of increments in "life time in good health" depending on its absolute value**

Introduction and Basic Idea

- **A typical representation of a loss function *L*:**

$$L(\lambda_1, \lambda_2, ..., \lambda_n, \mu_1, \mu_2, ..., \mu_n) = c(\lambda_1, \lambda_2, ..., \lambda_n, \mu_1, \mu_2, ..., \mu_n) + \frac{1}{\gamma}\sum_{i=1}^{n} \lambda_i \mu_i - \frac{g}{\gamma}$$

where

g : net gain per time unit

n : number of categories of loss generating events

λ_i : intensity of loss generating event I

μ_i : monetary loss associated with loss generating event i

γ : interest rate

Normalization of the loss function by average current salary rate transforms the loss function into time units and accounts for inflation and purchasing power variation over time

Equilibrium of Work/Free Time in Good Health

- **Some definitions:**

  r : **fraction of time in good health**

  w : **fraction of time used for work**

  G : **GDP within a certain geographical domain**

  S : **average salary within a certain geographical domain**

  p : **"time equivalent productivity"** $p = \frac{G}{S}$

  It follows: $G = pS = pw(S/w)$

  with S/w **being the salary per work time unit**

Equilibrium of Work/Free Time in Good Health

- **The concept of perceived change is introduced in terms of "worth" is introduced as the dimensionless ratio of the "change" to the "present" (Weber-Fechner Law)**

The worth of an increment in work time w is thus measured as:

$$\frac{\mathrm{d}(pw)}{pw}$$

The worth of an increment in free time in good health *(r-w)*:

$$\frac{\mathrm{d}(r-w)}{(r-w)} = \frac{\mathrm{d}(1-w/r)}{(1-w/r)}, \quad \text{for constant } r$$

Equilibrium of Work/Free Time in Good Health

- **Since** $\dfrac{\mathrm{d}(1-w/r)}{(1-w/r)} < 0$ for positive dw

an *equilibrium equation* may be formulated between p and w for constant worth:

$$c(r)\frac{\mathrm{d}(pw/r)}{pw/r}+[1-c(r)]\frac{\mathrm{d}(1-w/r)}{1-w/r}=0, \quad \text{for some constant } c(r) \text{ between 0 and 1}$$

assuming that p and w may be expressed as functions of each other – locally – and that the free market will try to maintain equilibrium we have

$$\left(\frac{c(r)}{w/r}-\frac{1-c(r)}{1-w/r}\right)\mathrm{d}(w/r)=-\frac{c}{p}\mathrm{d}p, \quad \text{relating } w/r \text{ with } p$$

Equilibrium of Work/Free Time in Good Health

- **It is noticed that** $\mathrm{d}p = 0 \;\; \text{for } w/r = c(r)$

  and by sign analysis that $p \;\; \text{is minimal for } w/r = c(r)$

- **The solution to the equilibrium equation is:**

$$p(w/r, r) = p_{\min}(r) \frac{(1 - w/r)^{1-1/c(r)} (w/r)^{-1}}{(1 - c(r))^{1-1/c(r)} c(r)^{-1}}$$

with

$$p_{\min}(r) = \arg\min_{w} \; p(w/r, r)$$

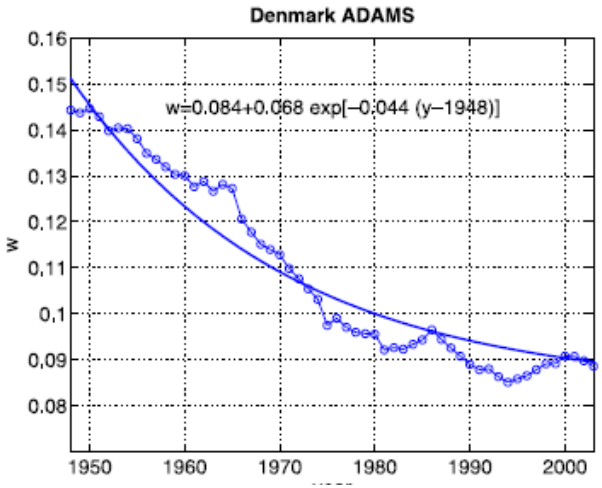
Equilibrium of Work/Free Time in Good Health

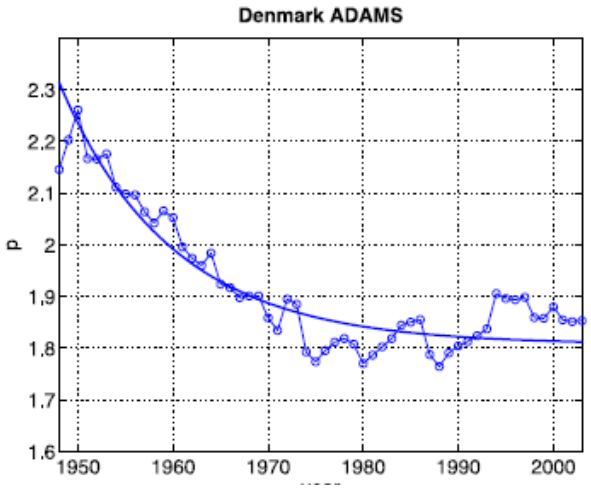
- **The relation**

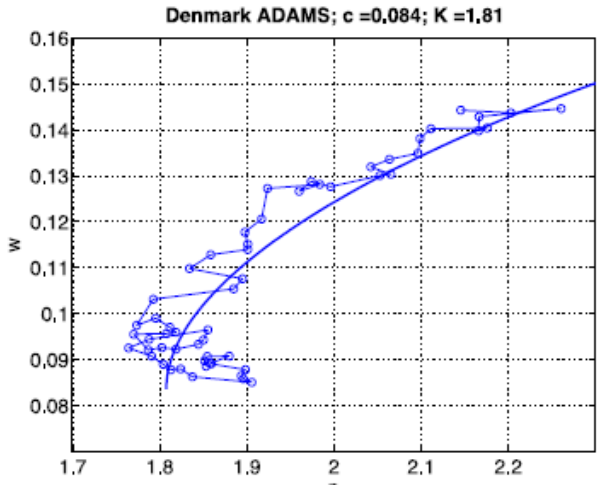
$$p(w/r,r) = p_{\min}(r)\frac{(1-w/r)^{1-1/c(r)}(w/r)^{-1}}{(1-c(r))^{1-1/c(r)}c(r)^{-1}}$$

$$p_{\min}(r) = \arg\min_{w} p(w/r,r)$$

may be supported by data (more in next presentation)







$w = c + ae^{-b(t-1948)} \to c$ as $t \to \infty$ with parameters estimated by joint least squares analysis for $r = 1$

Equilibrium of Work/Free Time in Good Health

- **The relation**

$$p(w/r,r) = p_{\min}(r)\frac{(1-w/r)^{1-1/c(r)}(w/r)^{-1}}{(1-c(r))^{1-1/c(r)}c(r)^{-1}}$$

$$p_{\min}(r) = \arg\min_{w} p(w/r,r)$$

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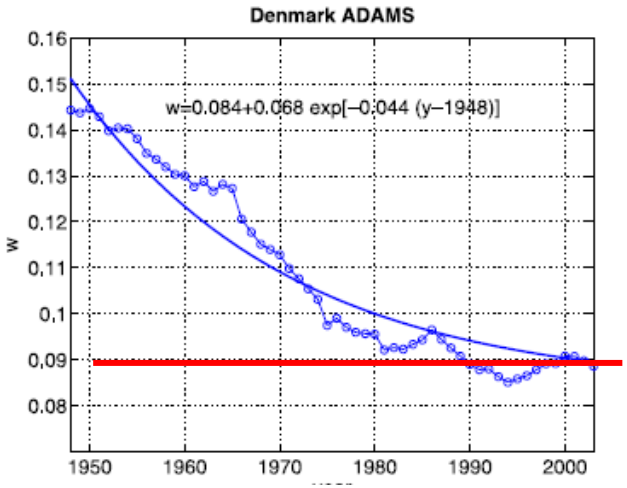

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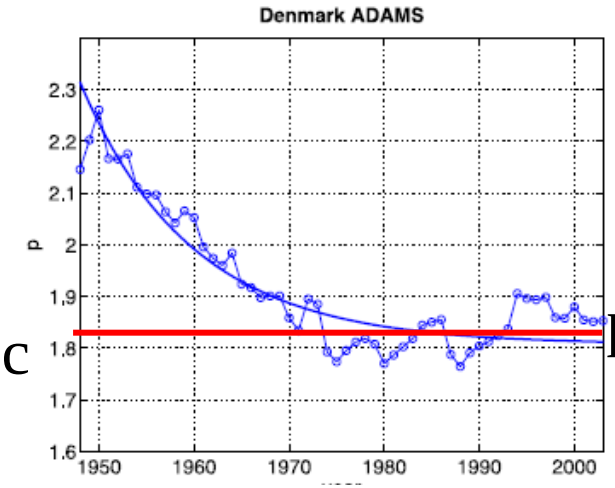
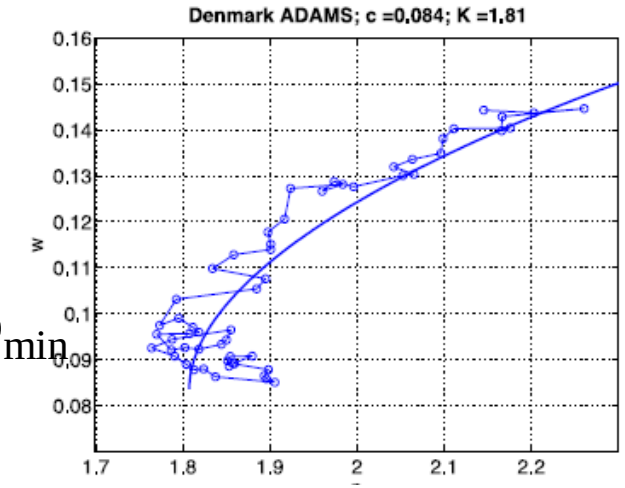


$p_{\min}$



$w = c + ae^{-b(t-1948)} \to c$ as $t \to \infty$ with parameters estimated by joint least squares analysis for $r = 1$

Equilibrium of Work/Free Time in Good Health

- **The actual value of r, i.e. r_0 can be determined from population health statistics, r_0 = 0.9 may be appropriate**

- **If the equilibrium equation does not yield zero there is an unbalance between the work efforts and the available free time**

$$c(r)\frac{\mathrm{d}(pw/r)}{pw/r}+[1-c(r)]\frac{\mathrm{d}(1-w/r)}{1-w/r}=0, \quad \text{for some constant } c(r) \text{ between 0 and 1}$$

  - **for a positive value the living standards are increasing**
  - **for a negative value the living standards are decreasing**

Equilibrium of Work/Free Time in Good Health

- **Thus we may establish (relatively) a non-dimensional *wealth index* as:**

$$\frac{\mathrm{d}I_r}{I_r} = V(r)\left(c(r)\frac{\mathrm{d}(pw)}{pw/r} + ([1-c(r)]\frac{\mathrm{d}(1-w/r)}{1-w/r} \right)$$

- **Integration yields**

$$I_r = J(r)\left[\left(p\frac{w}{r} \right)^{c(r)} \left((1-\frac{w}{r} \right)^{1-c(r)} \right]^{V(r)} =$$

$$J(r)\exp\left\{ V(r)\left[c(r)\log\left(p\frac{w}{r} \right) + \left[1-c(r)\right]\log\left(1-\frac{w}{r} \right) \right] \right\}$$

The Invariance Principle

- **Consider now an increment in the expected value of life in good health** $\mathrm{d}(rE)$ **by some risk reducing measure**

  A part of this is used for work i.e.: $(w/r)\mathrm{d}(rE)$ **and has the experience worth:**

  $$(pw/r)\mathrm{d}(rE)/(pwE) = \mathrm{d}(rE)/(rE)$$

  the rest is free time with worth:

  $$(1-w/r)\mathrm{d}(rE)/[(1-w/r)rE] = \mathrm{d}(rE)/(rE)$$

  The previously derived combination rule may be applied again yielding:

  $$\mathrm{d}(rE) = V(r)\frac{\mathrm{d}(rE)}{rE} = V(r)\left[\frac{\mathrm{d}r}{r} + \frac{\mathrm{d}E}{E}\right]$$

The Invariance Principle

- ***The invariance principle*** **postulates that the perceived increment (loss) of p caused by a measure of risk reduction should be balanced by the perceived increment (gain) in the expected value of life in good health, i.e.:**

$$\frac{\mathrm{d}I_r}{I_r} + V(r)\left[\frac{\mathrm{d}r}{r} + \frac{\mathrm{d}E}{E}\right] = 0$$

$$\Downarrow$$

$$-\frac{\mathrm{d}p}{p} = \frac{1}{c(r)}\frac{\mathrm{d}E}{E} + \left[\frac{1-c(r)}{r-w} + c'(r)\log\left(\frac{pw}{r-w}\right)\right]\frac{\mathrm{d}r}{c(r)}$$

for actual values of r and stationary state of w, i.e. r_0 and c we have

$$-\frac{\mathrm{d}p}{p_{\min}} = \frac{r_0}{c}\frac{\mathrm{d}E}{E} + \left[\frac{1}{c} + \frac{r_0}{c}c'(r_0)\log\left(\frac{p_{\min}c}{r_0 - c}\right)\right]\mathrm{d}r$$

The Invariance Principle

- **In order to apply** $$-\frac{\mathrm{d}p}{p_{\min}} = \frac{r_0}{c}\frac{\mathrm{d}E}{E} + \left[\frac{1}{c} + \frac{r_0}{c}c'(r_0)\log\left(\frac{p_{\min}c}{r_0 - c}\right)\right]\mathrm{d}r$$

  it is needed to assess $c'(r_0)$

  which is not achievable from data – why the normative choice is made that $c(r) = c/r$ **using the postulate that** p **is not influenced by** r **(can be relaxed to be valid only for small variations of** r **close to** r_0**)**

  This yields

$$-\frac{\mathrm{d}p}{p_{\min}} = \frac{r_0}{c}\frac{\mathrm{d}E}{E} + \left[\frac{1}{c} - \frac{1}{r_0}\log\left(\frac{p_{\min}c}{r_0 - c}\right)\right]\mathrm{d}r$$

The Invariance Principle

- **The Life Quality Time Allocation Index (LQTAI) is associated with the term:** $I_r rE$
- **The fraction of the time equivalent productivity p_{min} which should be invested into avoiding that an event causes the expected life to decrease and the expected life at good health to decrease may be assessed from the two right side terms of**

$$-\frac{\mathrm{d}p}{p_{\min}} = \frac{r_0}{c}\frac{\mathrm{d}E}{E} + \left[\frac{1}{c} - \frac{1}{r_0}\log\left(\frac{p_{\min}c}{r_0 - c}\right)\right]\mathrm{d}r$$

  - **normalizing by the mean accident rate yields real time values**
  - **multiplying with the average salary transforms to monetary units, ICAF and ICAI, respectively**

The Invariance Principle

- **The result**

$$-\frac{\mathrm{d}p}{p_{\min}} = \frac{r_0}{c}\frac{\mathrm{d}E}{E} + \left[\frac{1}{c} - \frac{1}{r_0}\log\left(\frac{p_{\min}c}{r_0 - c}\right)\right]\mathrm{d}r$$

can also be achieved directly through

$$\frac{\mathrm{d}\left(I_r rE\right)}{I_r rE} = 0, \text{ for constant } w$$

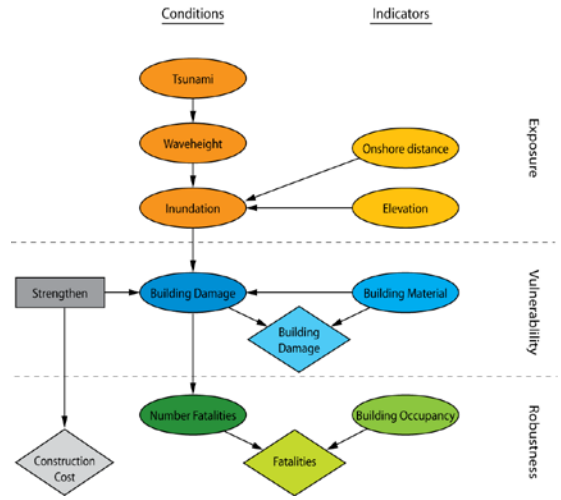
the result may be applied for arbitrary hazards ranging from food poisoning, over natural hazards to e.g. emissions of toxic gases/substances – the difference only concerns the assessment of $\mathrm{d}p$, $\mathrm{d}E$ and $\mathrm{d}r$

Conclusions

- **The LQTAI formulation can be considered a generalization of the LQI**
- **All value assessments are transformed to time equivalents**
- **The derivation of the LQTAI follows a different philosophy than that of the LQI**
- **The LQTAI and the LQI yields almost identical results**
- **The LQTAI may be applied for also for a non-stationary macro-economy with varying work time ratio**

Some Personal Observations

- **The LQTAI derivation rely on the concept of worth in modeling the perceived value of increments in productivity, life expectancy and expectancy of life in good health**
- **for the perception for individuals in case of monetary increments this is clearly a valid concept (Kahneman and Tversky)**
- **for an aggregation over a "society" this concept could be applied for monetary increments – but if the distribution of wealth is rather skew – the perception of wealth would not be justly represented**
- **A question also concerns whether increments in other attributes such as safety to life and health are perceived relatively in the same manner as increments in monetary units**



Thanks for your attention !