Deriving target reliabilities from the Life Quality Index

Katharina Fischer,
Celeste Barnardo-Viljoen & Michael H. Faber

ETH Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zürich

ibk Institut für Baustatik und Konstruktion
Institute of Structural Engineering

Background: JCSS target reliabilities for structural design

- The target reliabilities in the JCSS Probabilistic Model Code were derived based on monetary optimization studies by Rackwitz.
- Human consequences in case of failure are taken into account only qualitatively (or quantitatively in monetary terms).

⇨ Derive "minimum" target reliabilities based on the LQI criterion

⇨ Discuss interaction with JCSS target reliabilities / optimization

Relative cost of safety measure	Consequences of failure		
	Minor	Moderate	Large
Large (A)	$\beta = 3.1\left(P_f \approx 10^{-3}\right)$	$\beta = 3.3\left(P_f \approx 5 \cdot 10^{-4}\right)$	$\beta = 3.7\left(P_f \approx 10^{-4}\right)$
Normal (B)	$\beta = 3.7\left(P_f \approx 10^{-4}\right)$	$\beta = 4.2\left(P_f \approx 10^{-5}\right)$	$\beta = 4.4\left(P_f \approx 5 \cdot 10^{-6}\right)$
Small (C)	$\beta = 4.2\left(P_f \approx 10^{-5}\right)$	$\beta = 4.4\left(P_f \approx 5 \cdot 10^{-6}\right)$	$\beta = 4.7\left(P_f \approx 10^{-6}\right)$

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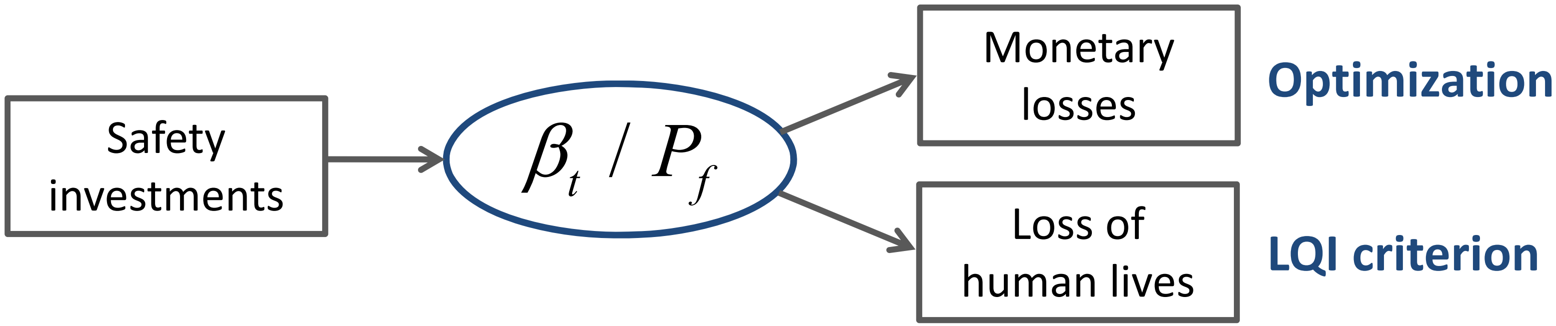


Monetary optimization: Rackwitz' renewal theoretic approach

Objective function (simplified):

$$\min_p \left\{ C(p) + A(p) + D(p) \right\} =$$

$$\min_p \left\{ C(p) + C(p) \cdot \omega / \gamma + \left(C(p) + H \right) P_f(p) / \gamma \right\}$$

constr. costs | obsolescence costs | damage / failure costs

Limit state function & safety costs (p central safety factor)

$$P_f(p) = P[R - S < 0] \qquad p = E[R]/E[S] \qquad C(p) = C_0 + C_1 p$$

⇨ C_1/C_0 "Relative costs of safety measure" (also: ω, γ)
H/C_0 "Consequences of failure"

Basing the LQI criterion on the same assumptions

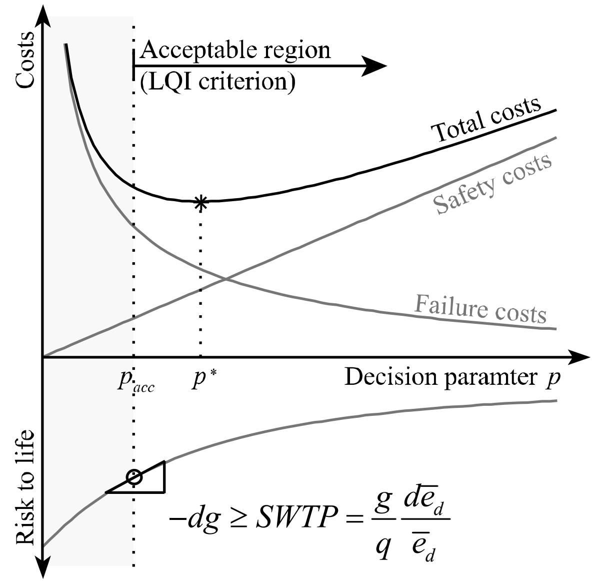
$$\frac{d\left(C(p)+A(p)\right)}{dp} \geq -\frac{g}{q} C_x \cdot \frac{d\mu(p)\cdot N_P}{dp}$$

Marginal life saving costs = safety investments

SWTP for mortality reduction $d\mu$

$$C_1\left(1+\omega/\gamma_S\right) \geq -\frac{1}{\gamma_S}\cdot\frac{g}{q} C_x \cdot N_F \cdot \frac{dP_f(p)}{dp}$$

$$\Rightarrow \boxed{-\frac{dP_f(p)}{dp} \leq \frac{C_1\left(\gamma_S+\omega\right)}{g/q\cdot C_x\cdot N_F} = K_1}$$



$$\frac{\text{Marginal safety costs}}{\text{Monetized human consequences}}$$

Estimating the K_1 constant based on simplified assumptions

(a) Office Building, $C_0 = 2'000$ CHF/m^2			
N_F/m^2	C_1 / C_0		
	0.001	**0.01**	**0.1**
0.0001	2E-04	2E-03	2E-02
0.001	2E-05	2E-04	2E-03
0.01	2E-06	2E-05	2E-04
0.1	2E-07	2E-06	2E-05

(b) Bridge, $C_0 = 10$ Mio.CHF			
N_F	C_1 / C_0		
	0.001	**0.01**	**0.1**
0.1	1E-03	1E-02	1E-01
1	1E-04	1E-03	1E-02
10	1E-05	1E-04	1E-03
100	1E-06	1E-05	1E-04

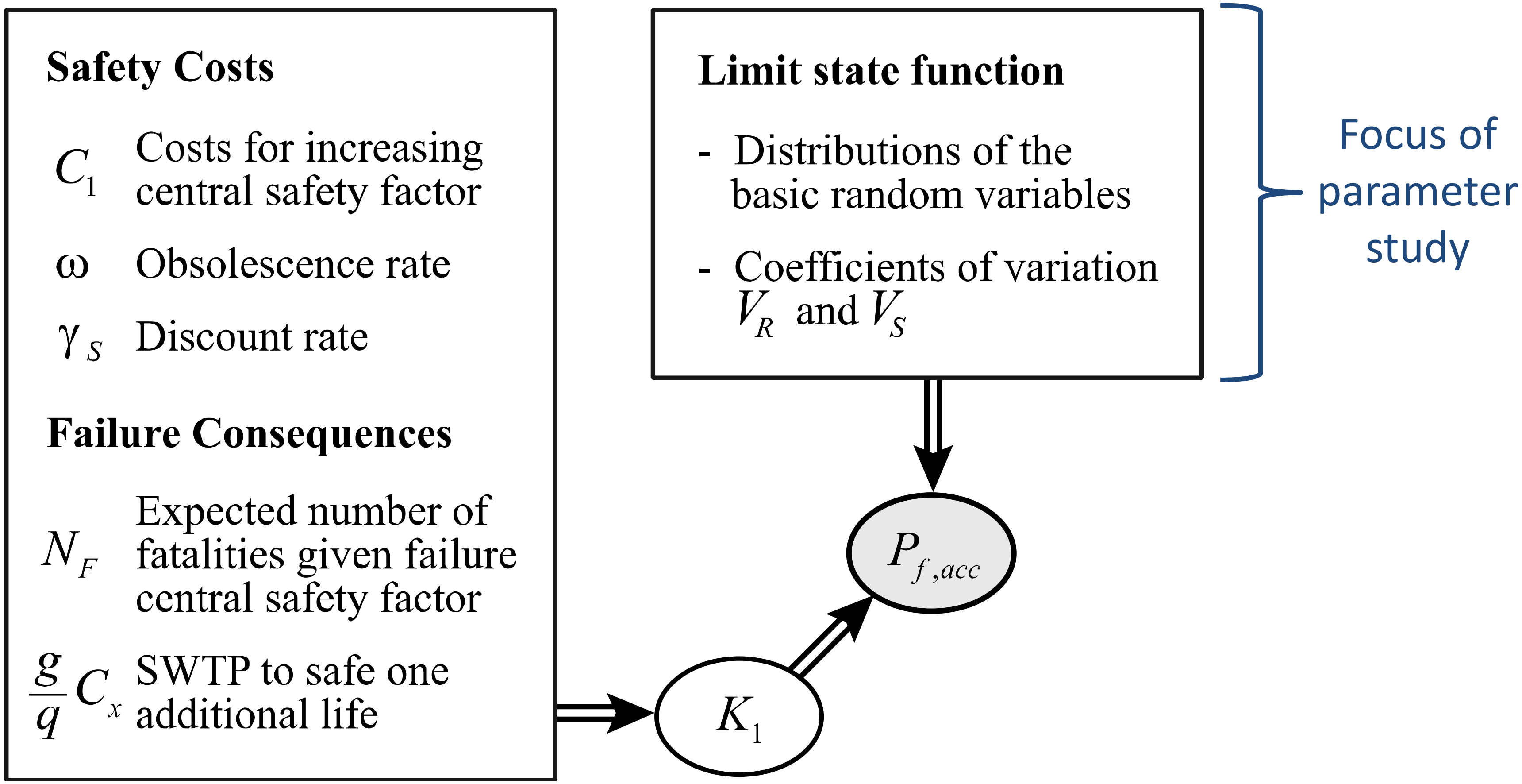
Assumptions: $g/q \cdot C_x = 5.1 Mio.CHF \quad \omega = 0.02 \quad \gamma_S = 0.03$

$$\Rightarrow \boxed{ -\frac{dP_f(p)}{dp} \leq \frac{C_1(\gamma_S + \omega)}{g/q \cdot C_x \cdot N_F} = K_1 }$$

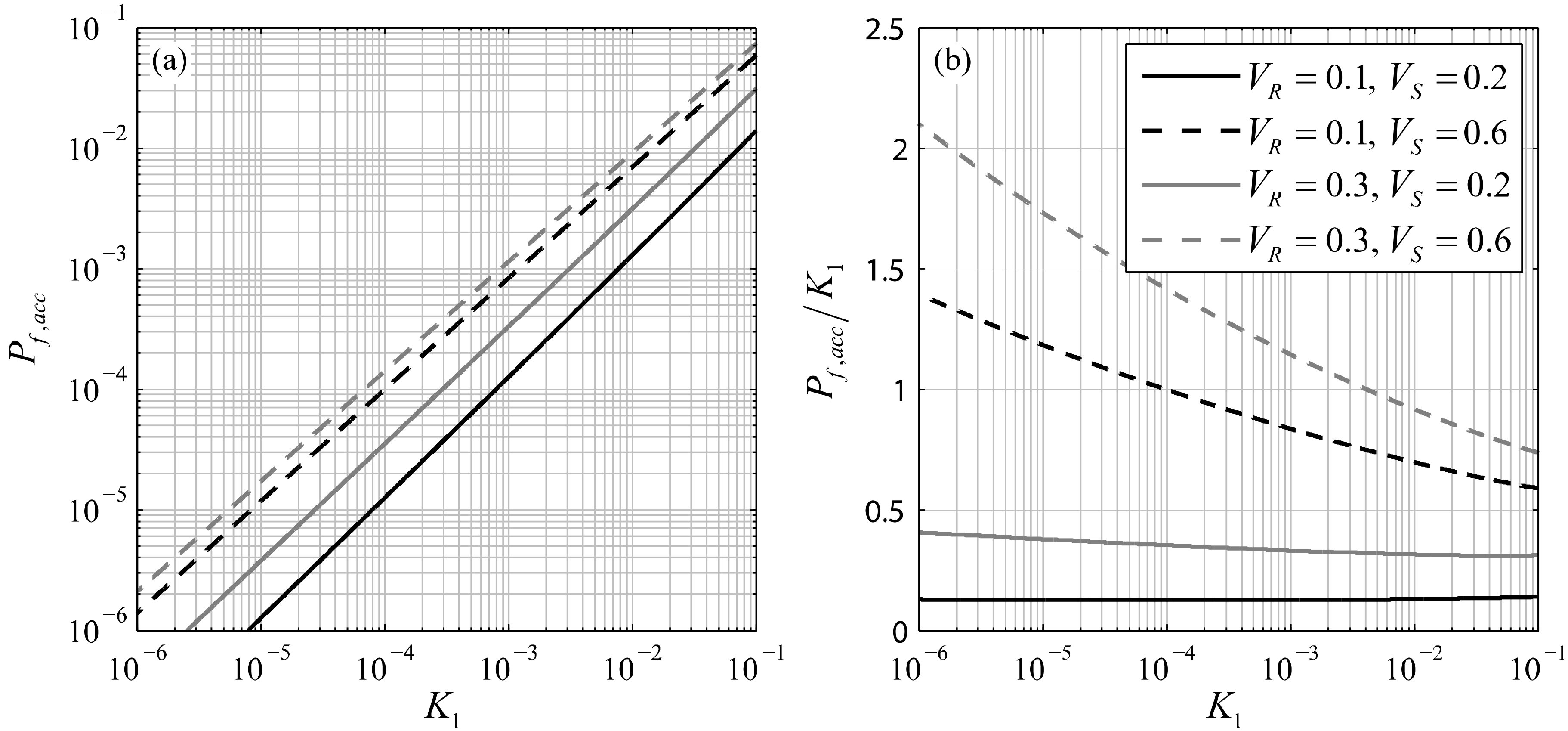
$$\frac{\text{Marginal safety costs}}{\text{Monetized human consequences}}$$

Parameter study

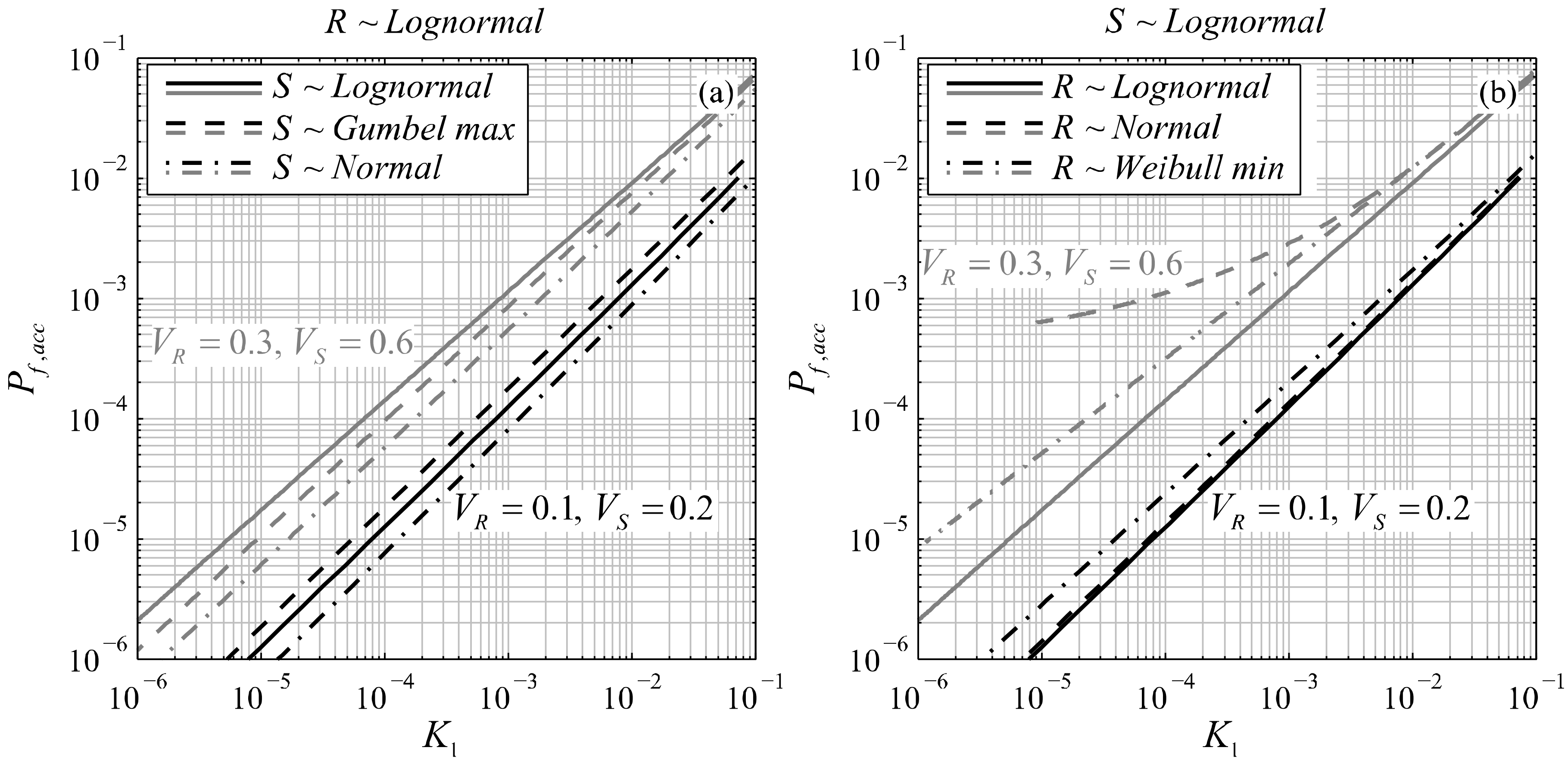
Parameters / assumptions influencing the LQI target reliability



Influence of cost / consequence constant K_1



Influence of distributional assumptions



Interaction with monetary optimization

$$d\left(C\left(p^*\right)+A\left(p^*\right)+D\left(p^*\right)\right)\Big/dp=0$$

Optimality condition

$$\frac{d\left(C\left(p\right)+A\left(p\right)\right)}{dp}\geq-\frac{g}{q}C_x\cdot\frac{d\mu\left(p\right)\cdot N_P}{dp}$$

Acceptance criterion

$$P_f\left(p^*\right)<<\gamma$$

General criterion

$$\underbrace{\frac{g/q\cdot C_x\cdot N_F}{C\left(p^*\right)+H}}_{?}\leq\underbrace{\frac{\gamma_S+\omega}{\gamma+\omega}}_{\leq 1}$$

or

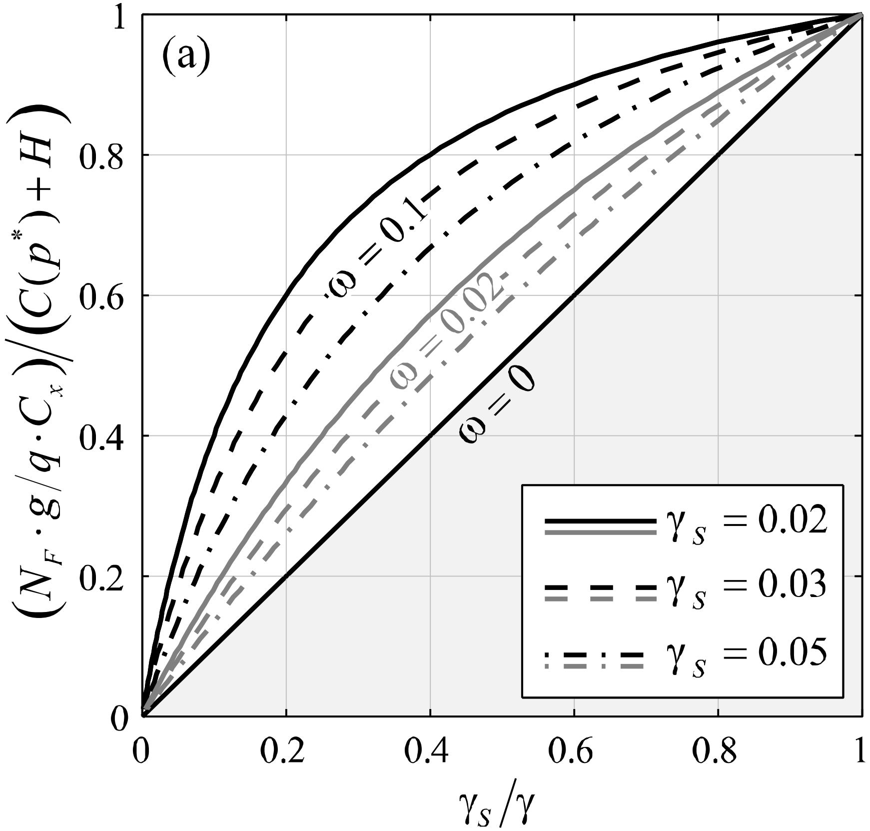
With compensation costs

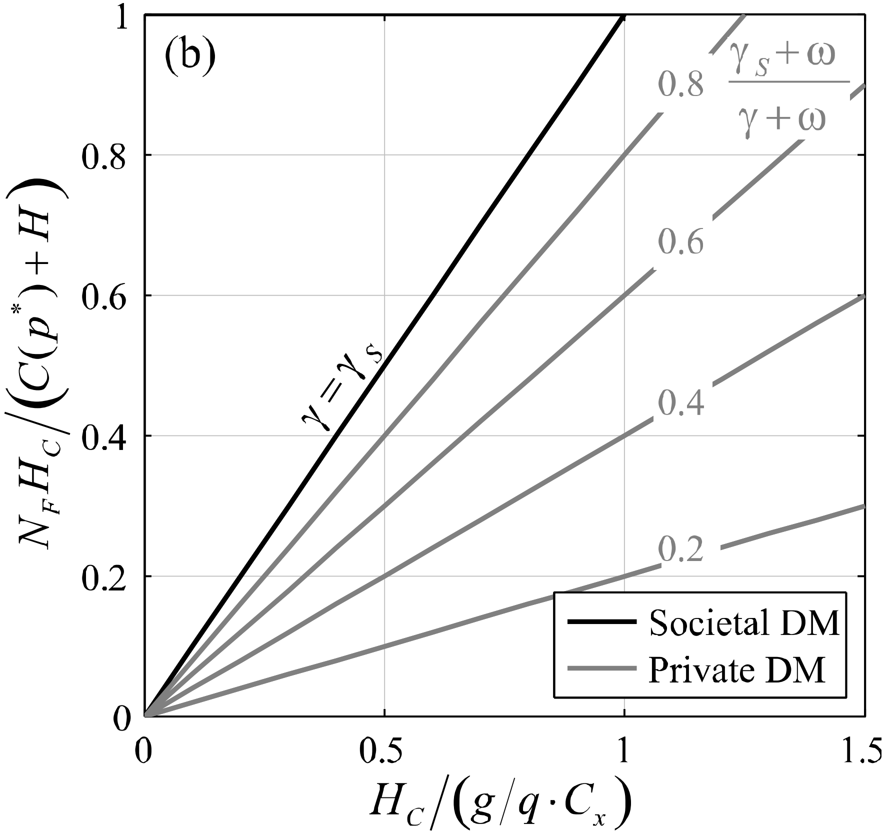
$$\underbrace{\frac{N_F H_C}{C\left(p^*\right)+H}}_{\leq 1}\leq\underbrace{\frac{\gamma_S+\omega}{\gamma+\omega}}_{\leq 1}\cdot\underbrace{\frac{H_C}{g/q\cdot C_x}}_{?}$$

Interaction LQI - optimization

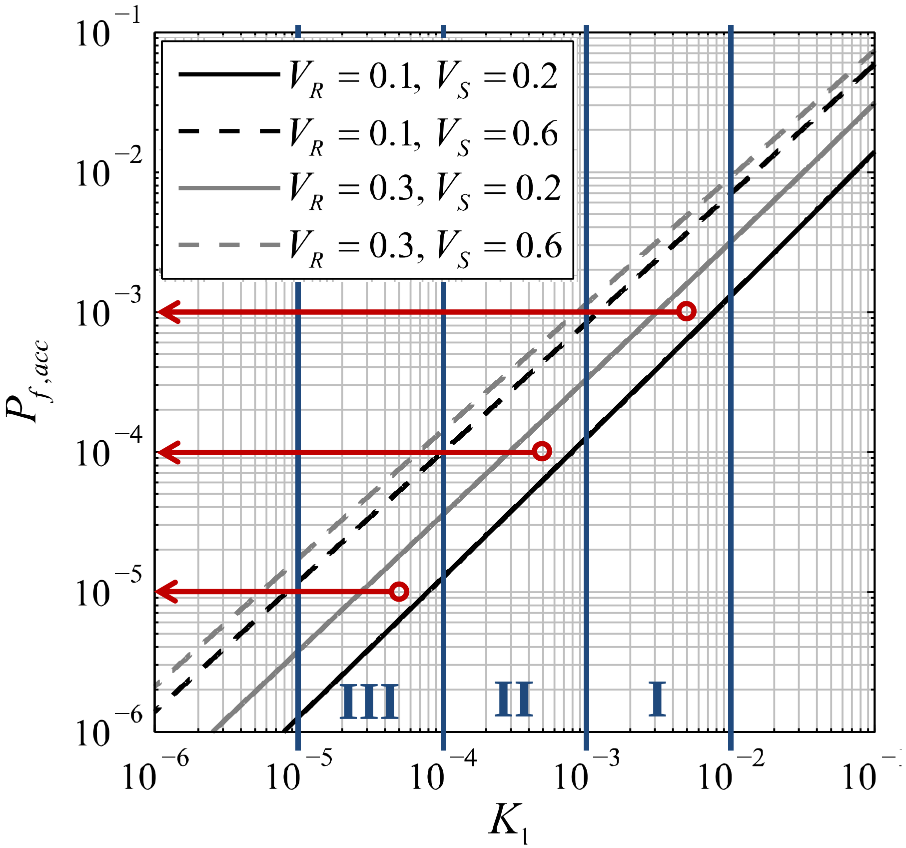
$$\frac{g/q \cdot C_x \cdot N_F}{C\left(p^*\right)+H} \leq \frac{\gamma_S+\omega}{\gamma+\omega}$$

$$\frac{N_F H_C}{C\left(p^*\right)+H} \leq \frac{\gamma_S+\omega}{\gamma+\omega} \cdot \frac{H_C}{g/q \cdot C_x}$$





Derivation of a simplified LQI target reliability format



Simple rule for $0.1 \le V \le 0.3$:

$$P_{f,acc} \approx \frac{K_1}{5} = \frac{1}{5} \cdot \frac{C_1(\gamma_S + \omega)}{g/q \cdot C_x \cdot N_F}$$

Relative life saving costs	Range for K_1 constant	LQI target reliability
Large (I)	$10^{-3} \div 10^{-2}$	$\beta = 3.1 \left(P_f \approx 10^{-3}\right)$
Medium (II)	$10^{-4} \div 10^{-3}$	$\beta = 3.7 \left(P_f \approx 10^{-4}\right)$
Small (III)	$10^{-5} \div 10^{-4}$	$\beta = 4.2 \left(P_f \approx 10^{-5}\right)$

Discussion & Conclusion

The LQI target reliabilities in practice

- The LQI target reliabilities define a societal boundary condition for monetary optimization.
- Simple rules for checking whether LQI becomes active.
- The JCSS target reliabilities may be used as a substitute for direct optimization (caution: different meaning of cost classes!).
- To be defined: A range for the expected number of fatalities given failure within which the LQI target reliabilities may be used.

Open questions

- Cost assessment easiest at component level (limit state design), Consequence assessment at structural level.
- Interaction LQI / optimization in the context of regulation.