JCSS PROBABILISTIC MODEL CODE
PART 3: RESISTANCE MODELS

April 2011

3.12 FATIGUE MODELS FOR METALLIC STRUCTURES

Table of contents:

List of symbols (main text)

a	crack depth
c	half-length of the crack at the surface
$f( )$	probability density function, resistance interaction criterion
$g( )$	limit state function
m	parameter in the S-N-approach
m	parameter in the Paris Law
n	number of actual stress cycles
t	time
A	Paris Law parameter
C	model uncertainty
B	plate thickness
D	damage according to Miner's Rule
$E[ ]$	expectation operator
K	parameter in the S-N-model
K	elastic stress intensity factor in the fracture mechanics model
K_{mat}	material fracture toughness
K_r	fracture ratio
K_{res}	stress intensity factor for residual stresses
K_s	stress intensity factor for the applied total stress S
L_r	plastic collapse ratio
M_k	stress intensity magnification factor accounting for the stress concentration due to the weld geometry
N	number of stress cycles to failure at a constant amplitude stress range ΔS
R_S	stress ratio (minimum stress divided by maximum)
R	normalised fracture toughness resistance parameter
T_{min}	minimum operating temperature
T_0	temperature variability of T_{27J}
T_{27J}	temperature corresponding to a CVN of 27J
X	random variable vector
Y	stress intensity correction factor

λ	Weibull shape parameter
σ	Weibull scale parameter
ε	statistical error in S-N curve
ρ	plasticity correction factor
η	parameter in fracture criterion
ξ	thickness correction exponent
σ	stress, including stress concentrations due to the joint geometry
σ_{net}	net section stress (function of the crack size)
σ_{res}	residual stresses
σ_y	yield stress
Δ	range operator
Φ	complete elliptic integral of the second kind

Subscripts:

cr	limiting or critical value
o	initial value
tr	transition value
0	threshold value
glob	global stress analysis
scf	stress concentration factor
sif	stress intensity factor
n	number of cycles
m	membrane
b	bending
ref	reference value

3.12.1 Scope

This chapter is concerned with the treatment of fatigue in metallic structures with particular emphasis on welded joints. The document is applicable to structures such as buildings, bridges, offshore structures, masts, cranes etc., in which the fatigue damage is caused by the cyclic action of wind, waves, traffic or mechanical vibrations. Low-cycle fatigue, which may occur for example due to an earthquake, is not treated.

In this document three different approaches to the formulation of the fatigue limit state will be considered, respectively based on:

(1) *S*-*N* lines in combination with Miner's Damage accumulation rule;
(2) a fatigue crack growth model;
(3) a fatigue crack growth model in combination with fracture resistance.

The models presented to not include the situation following a crack through situation of the wall thickness. The fatigue loading is characterised by the number of stress cycles and the magnitude of stress range for each cycle. A probabilistic description of stress ranges can be developed if the stochastic properties of the stress process over the entire lifetime of the structure are known. For some models the expectation of the m^{th} moment of the arbitrary point in time distribution of the stress is sufficient.

This chapter is based on BS 7910:2005 as far as possible.

3.12.2 S-N-approach

Damage model

An S-N curve is a relation between the stress range under constant amplitude loading and the number of stress cycles to failure. The standard S-N curve can be expressed in the form of:

$$N \Delta\sigma^m = K \tag{2.1}$$

where N is the number of stress cycles to failure at a constant amplitude stress range $\Delta\sigma$, K and m are the material parameters. In order to deal with variable amplitude loading in the S-N approach, fatigue damage is quantified in terms of Miner's damage summation. According to this rule all stress cycles cause proportional fatigue damage, which is linearly additive. For an ergodic variable stress process, the scatter in the stress history may be neglected and the damage D_n due to n cycles is given by:

$$D_n = E(n)\left[\frac{1}{K} E\left[\Delta\sigma^m\right]\right] \tag{2.2}$$

E[..] denotes the expectation operator. Some elaborations are given in Annex B. According to this model failure occurs nominally when D_n is equal to unity.

In most cases a model with two lines is being used, having parameters K_1, K_2, m_1 and m_2. The stress range level $\Delta\sigma_{tr}$ at which the two lines intersect (transition point) is defined by:

$$\Delta\sigma_{tr} = \left\{\frac{K_2}{K_1}\right\}^{1/(m_2-m_1)} \tag{2.3}$$

In case of two branches (2.2) is transformed into:

$$D_n = E(n)\left[\frac{1}{K_1} E\left[\Delta\sigma^{m_1}\right]_{\Delta\sigma_0}^{\Delta\sigma_{tr}} C_{glob}^{m_1} C_{scf}^{m_1} + \frac{1}{K_2} E\left[\Delta\sigma^{m_2}\right]_{\Delta\sigma_{tr}}^{\infty} C_{glob}^{m_2} C_{scf}^{m_2}\right] \tag{2.4}$$

The expectation operator *E[..]* in this case is defined by:

$$E\left[\Delta\sigma^m\right]_A^B = \int_A^B s^m f_{\Delta\sigma}(s)ds \tag{2.5}$$

where $f_{\Delta\sigma}(s)$ is the probability density function of the stress ranges $\Delta\sigma$. The integration limit $\Delta\sigma_0$ represents the fatigue limit (if present) and $\Delta\sigma_{tr}$ the transition point defined by (2.3). Further, in this approach C_{glob} and C_{sif} have been added as the model uncertainties in the global and local stress analysis, respectively.

The effects of weld geometry, residual stresses and through thickness stress variation are implicitly included in the values of K and m. The effect of factors such as, plate thickness, environment, weld toe grinding and post-weld heat treatment, etc. is accounted through appropriate corrections to the basic S-N curve. For instance, a plate thickness correction factor may be given by:

$$a_{cor} = \left[\frac{B}{B_{ref}}\right]^{m\xi} \tag{2.6}$$

where ξ is the thickness correction exponent on stress, B is the thickness of the plate and B_{ref} is the reference plate thickness.

Limit state formulation for SN-approach:

The limit state function may be given by:

$$g(\boldsymbol{X}, t) = D_{cr} - D_{n} \tag{2.7}$$

where $\boldsymbol{X}$ is the vector of random variables, t the time, D_{cr} is Miner's Damage sum at failure, which may deviate systematically and randomly from 1.0 and D_{n} is the damage due to n cycles.

3.12.3 Fracture Mechanics Approach

Crack Growth model

The fatigue crack growth model is the bi-linear version (see Figure 3.1) of the model by Paris & Erdogan (1963):

$$\frac{da}{dN} = A_1(\Delta K)^{m_1} \quad \text{for } \Delta K_0 < \Delta K \le \Delta K_{tr}$$

$$\frac{da}{dN} = A_2(\Delta K)^{m_2} \quad \text{for } \Delta K_{tr} < \Delta K \qquad (3.1)$$

where a is the crack depth, da/dN is the instantaneous crack propagation rate and ΔK is the stress intensity factor (SIF) range at the crack tip, ΔK_0 a threshold value below which the crack is assumed to be non-propagating; A_i and m_i are material constants which can be determined from experiments. ΔK_{tr} is the value at the transition point.

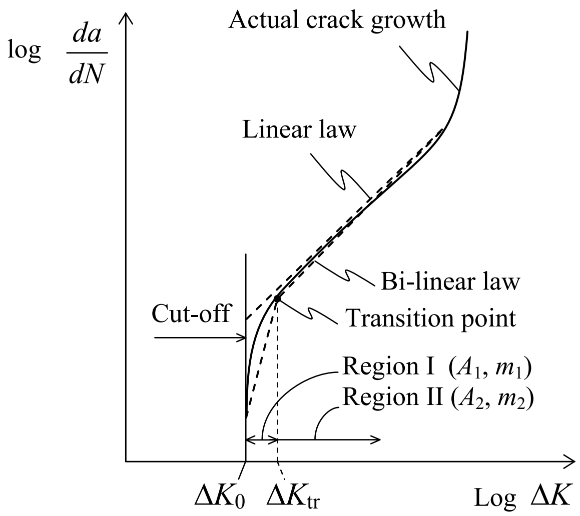


Fig. 3.1: Schematic representation of typical crack growth rates with linear and bi-linear approximations.

The stress intensity factor K for a given applied (fluctuating) stress σ is given as:

$$K = \sigma Y \sqrt{\pi a} \qquad (3.2)$$

The stress σ should be determined from the applied loading in the vicinity of the crack location (but not influenced by the crack or the details of the weld). It does, however, include the stress concentration, resulting from the global geometry of the joint. The stress intensity correction factor Y depends on the crack size normalised with respect to other relevant local dimensions (e.g. plate thickness) and the nature of stress distribution. For details on Y, see Annex A.

The through-thickness distribution of this stress is often assumed to consist of 'membrane' and 'bending' components and is characterised in terms of the ratio of the bending stress to the total stress, called 'degree-of-bending':

$$K = (\sigma_m Y_m + \sigma_b Y_b)\sqrt{\pi a} \qquad (3.3)$$

where the subscripts 'm' and 'b' refer to membrane and bending components respectively.

For welded joints micro-cracks often initiate from surface-breaking defects at the toe of the weld. These micro-cracks tend to coalesce to form a single, dominant fatigue crack of roughly semi-elliptical shape. Hence, semi-elliptical cracks in plated structures are of interest in many practical applications (figure 3.2). In this case, two crack dimensions, the depth *a* and the half-length at the surface *c,* become relevant both of which are functions of the fatigue loading process.

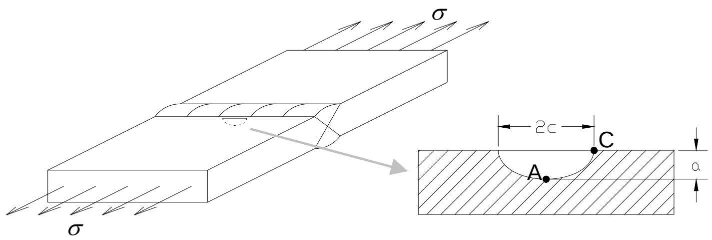


Figure 3.2: A semi-elliptical crack in a steel plate at/near the weld toe.

For each principal direction of crack growth a Paris type expression may be formulated,

$$\frac{da}{dN} = A\,(\Delta K_a)^m \quad \text{for } \Delta K_a > \Delta K_0$$
$$\frac{dc}{dN} = A\,(\Delta K_c)^m \quad \text{for } \Delta K_c > \Delta K_0 \qquad (3.4)$$

where the first expression relates to point A and growth in the depth direction, whereas the second expression relates to point C and growth in the length direction (see figure 3.2). For each of these points, the stress intensity factor range, ΔK_a and ΔK_c, is given by

$$\Delta K_a = \Delta\sigma\, Y_a \sqrt{\pi\, a}$$
$$\Delta K_c = \Delta\sigma\, Y_c \sqrt{\pi\, a} \qquad (3.5)$$

where $\Delta\sigma$ is the applied stress range and Y_a, Y_c are stress intensity correction factors for points A and C respectively.

Once equations (3.5) are substituted into (3.4), a pair of coupled differential equations is obtained. With the exception of the material parameters (*A* and *m*) and the applied stress range $\Delta\sigma$, all other terms are a function of the crack size (*a, c*), which clearly changes during the fatigue loading process.

Limit state formulations

Using the fracture mechanics approach, the time dependent limit state function can be formulated as

$$g(\boldsymbol{X}, t) = a_{cr} - a(t) \tag{3.6}$$

where $\boldsymbol{X}$ is the vector of random variables, a_{cr} is a limiting crack depth (for example plate thickness) and $a(t)$ is the crack depth after a service exposure of time t. At $t = 0$ the crack size has an initial value a_o. The calculation of the crack size at time t is not trivial as the stress is a random process. Assuming for $\Delta\sigma$ a sufficiently mixing (ergodic) process we may use the expectation of the stress range to find the expectation of da/dN (conditional upon the stress and fatigue model):

$$E(\frac{da}{dN}) = A_1 E[\Delta\sigma^{m_1}]_{\Delta\sigma_0}^{\Delta\sigma_{tr}} \ (C_{glob} C_{scf} C_{sif}\ Y_a \sqrt{\pi a})^{m_1} + \\ + A_2 E[\Delta\sigma^{m_2}]_{\Delta\sigma_{tr}}^{\infty} \ (C_{glob} C_{scf} C_{sif}\ Y_a \sqrt{\pi a})^{m_2} \tag{3.7}$$

where C are model uncertainty factors for the various stress calculations (global analysis, stress concentration and stress intensity factor) and:

$$E[\Delta\sigma^m]_A^B = \int_A^B s^m f_{\Delta\sigma}(s) ds \tag{3.8}$$

where $f_{\Delta\sigma}(s)$ is the probability density function of the stress ranges $\Delta\sigma$.

In general, the above integral is evaluated through an incremental numerical procedure, which involves sub-division of the total time into a number of steps. At each step, following the evaluation of equation (3.8) for the crack depth a , the second crack dimension c is computed as well, using a similar set of equations. In that case, however, the value Y_a is replaced by Y_c. As both a and c are simultaneously present in Y, the two differential equations are coupled.

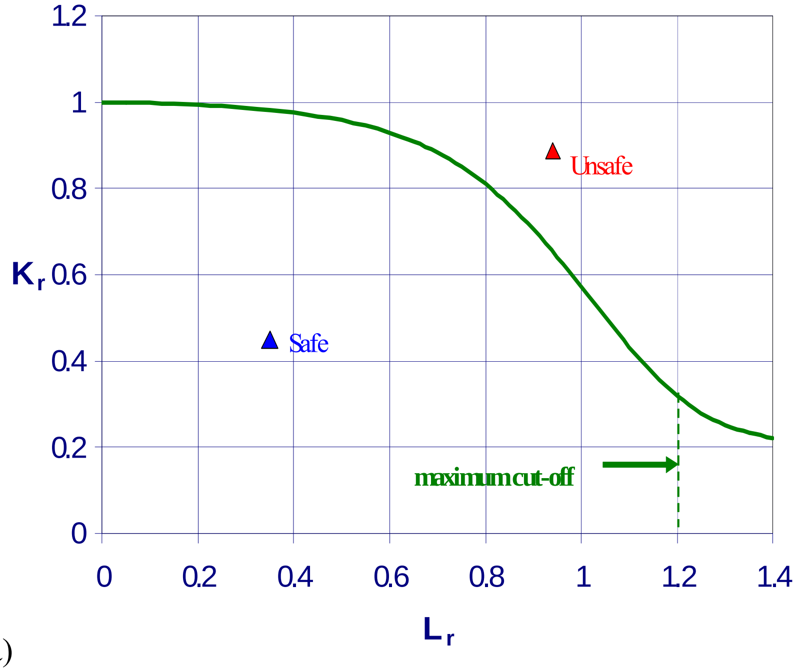
When the fatigue life of a fatigue sensitive detail is required, the critical crack size may for instance be taken as the plate thickness, but also as a critical value a_{cr} determined through the application of a fracture criterion. In that case the maximum crack size, a_{cr} that can be sustained by the component will depend on the material's yield strength and fracture toughness K_{mat}. The limit state function with respect to fracture can be formulated as:

$$g(\mathbf{X}, t) = \min_t f(K_r, L_r) \tag{3.9}$$

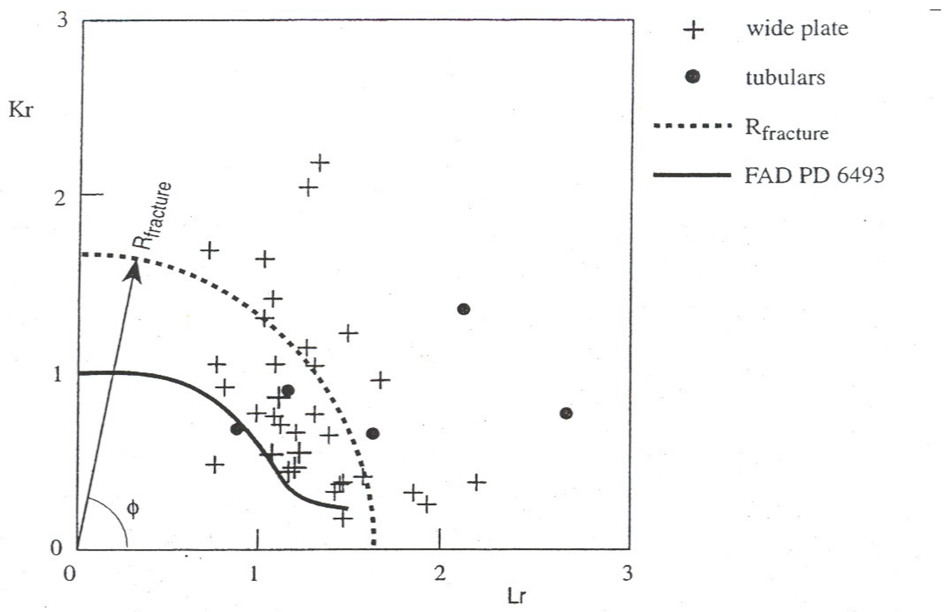
where $\boldsymbol{X}$ is the vector of random variables as a function of the time t, K_r is the fracture ratio, L_r is a measure of the proximity to plastic collapse and $f(\,)$ is an appropriate interaction criterion, as for example (see figure 3.3a) given in BS 7910 (BSI, 2005) or (see figure 3.3b) Dijkstra (1991) as:

$$f(K_r, L_r) = R - \sqrt{(K_r^2 + L_r^2)} \tag{3.10}$$

In this approach failure occurs when a stress cycle arrives that causes the reduced load bearing capacity of the cross section to be exceeded. The residual load bearing capacity depends on the actual crack size a and the interaction between the plastic and the brittle fracture failure modes. To evaluate the failure probability on the basis of (3.9) a time variant reliability method like the out-crossing approach is needed.



a)



b)

Figure 3.3 K_r - L_r interaction curves according to BS 7910 (above) and Dijkstra (below)

In (3.10) the variable R is a normalised resistance parameter (nominally equal to 1.0) and the quantities K_r and L_r are defined as follows:

$$K_r = \frac{K_I}{K_{mat}} + \rho \quad \text{and} \quad L_r = \frac{\sigma_{net}}{\sigma_y} \qquad (3.12)$$

where

K_{mat}	material toughness measured by stress intensity factor
K_I	stress intensity factor: $K_I = (Y\sigma)\sqrt{\pi a}$
ρ	plasticity correction factor
σ_{net}	net section stress (function of the crack size)

σ_y yield stress

The factor ρ may be determined by the following procedure. For $\chi = K_I^s L_r / K_I^p > 4$ an interpolating technique need to be applied (see BS 7910, 2005). For $\chi \leq 4$ we have:

$$\rho = \rho_1 \quad \text{for} \quad L_r \leq 0.8$$
$$\rho = 4 \rho_1 (1.05 - L_r) \quad \text{for} \quad 0.8 < L_r < 1.05 \tag{3.13}$$
$$\rho = 0 \quad \text{for} \quad L_r \geq 1.05$$

with $\rho_1 = 0.1\chi^{0.714} - 0.007 \chi^2 + 3 * 10^{-5} \chi^5$ (3.14)

The crack size *a* may be determined from the fatigue crack growth models described above. The plasticity correction factor ρ reflects the interaction between the applied load (giving rise to the stress intensity factor *K*) and the residual stresses (giving rise to the stress intensity factor K_I^s). Different residual stress profiles for various cracked geometries and restraint conditions may be assumed but their use implies that K_I^s would need to be evaluated using either finite element techniques or the weight function method. A simplified (and conservative) method would be to approximate the residual stress field via a linear stress field subtended from the surface and crack tip stress values. The corresponding K_I^s can then be obtained by superposition of the tensile and bending solutions for the geometry in question.

Further details of this approximation are given by Tada *et al.* (2000). If the bending solution is not known, in which case the previously mentioned procedure cannot be applied, the residual stress field may be assumed to be uniform. This approach will, in general, yield very conservative results for deep cracks and less conservative results for shallow cracks.

3.12.4 Probabilistic models

Table 3.1 gives an overview of all random variables and their probabilistic models. There are four blocks: one for the S-N parameters, one for the Fracture Mechanics parameters, one for the stress analysis and one for the Weibull stress process approximation. Additional commentary is given below:

S-N curve parameters

The stochastic model for the parameters in the S-N-based approach can be estimated as follows:

Step 1: For the fatigue detail considered, find (as good as possible) the corresponding detail and test data in the EC3 Background Document prEN 1993-1-9. RWTH, Aachen, 2002.

Step 2: Fit test data to a linear S-N-model: $\log N = \log C - m \log \Delta\sigma + \varepsilon$ where N is the number of cycles to failure with stress range $\Delta\sigma$, ε is normally distributed with expected value 0 and standard deviation σ_ε. The parameters log *C*, m and σ_ε are fitted to the test data using the Maximum Likelihood Method including run-outs (no-failure tests) in the likelihood.

Step 3: The statistical uncertainty related to the parameter fits can be estimated using the Hessian of the Loglikelihhod function, and included in a reliability analysis.

A bilinear S-N-model can be fitted in a similar way. Usually m can be assumed fixed to 3 or 5.

Miner's sum at failure

Due to the random nature of fatigue damage, Miner's damage sum at failure (D_{cr}) is not necessarily always exactly unity. It should therefore be treated as a random variable expressing the shortcomings of Miner's rule. Wirsching (1984) suggests that D_{cr} be modelled as a lognormal variable with a mean of unity and coefficient of variation of 0.3.

Crack growth parameters

When a simple Paris type crack propagation model is used, a random variable description for *A*, is usually adequate. In this approach, the crack growth parameter *A* in the Paris law, can be used to describe the scatter in crack propagation data. The parameter *A* will in general depend on the applied stress ratio (= σ_{min} / σ_{max}) and the environmental conditions and is well described by a lognormal model with coefficient of variation typically in the range of 0.3-0.4. In the table the option to use a two branch model has been opened. For a one branch model one uses only m_2 and A_2. When a cut-off to crack growth ΔK_0 is adopted, ΔK_0 is lognormally distributed with a mean of 140 Nmm$^{-3/2}$ and a coefficient of variation V=0.4 (Austen, 1983). In the case of freely corroding steels $\Delta K_0 = 0$.

Initial defect size

The distribution of defect sizes should be based on defects existing in a structure entering service and includes those defects that are considered acceptable according to quality control standards, as well as those that remain undetected during fabrication. Thus defect occurrence rate, the amount and quality of NDE used, should be taken into account in developing a distribution for weld defect size. From the limited data available, the mean value and standard deviation of weld defect depth a_0 can be estimated to be 0.15 mm and 0.10 mm for “sound” quality welds. Lognormal, exponential and Weibull distributions have been used by researchers to fit weld defect data. Similarly, initial defect aspect ratio, defined as the ratio of initial defect depth to defect semi-length (a/c) may also be modelled as a lognormal variable with a mean of 0.62 and coefficient of variation of 0.4, Kountouris and Baker (1989).

Fatigue crack propagation model

The fatigue crack propagation model involves a number of parameters, which are subject to uncertainty. It is convenient to express all sources of uncertainty through a single basic variable C_{sif}, which gives the ratio of stress intensity geometry factor obtained by experiment to that computed using the proposed model. The statistics for this variable can be developed by comparison with experimental compliance function curves. The factor C_{sif} can be modelled as a lognormal variable with a mean of unity and coefficient of variation typically in the range of 0.15-0.25. Lower variability may be used if the stress intensity factors are computed using finite element models such as line-spring and weight function models.

Fracture Toughness

For the fracture toughness K_{mat} a three-parameter Weibull distribution is proposed to describe fracture toughness related to cleavage fracture (Burdekin & Hamour, 2000; Lukić & Cremona, 2001; Righiniotis & Chryssanthopoulos, 2003):

$$F_{K_{mat}}(k) = 1 - \exp\left[-\left(\frac{k - K_0}{\sigma}\right)^{\lambda}\right] \tag{4.1}$$

where

λ is the shape parameter, taken as 4 on the basis of experiments (Wallin, 1984).

K_0 is the threshold parameter, recommended is 20 MPa $m^{1/2}$ (Burdekin & Hamour, 2000).

σ is the scale parameter and may be obtained from the following correlation attributed to Wallin (Burdekin & Hamour, 2000):

$$\sigma = \left\{11 + 77\left[\exp\left(\frac{T - T_{27J} + T_0}{52}\right)\right]\right\}\left(\frac{25}{B}\right)^{1/4} \quad (K_{mat} \text{ in MPa } m^{1/2}) \tag{4.2}$$

where:

T operating temperature (°C) (random FBC process: μ = 10 °C, σ= 8 °C, Δ = 10 days)

T_{27J} temperature (°C) corresponding to a CVN of 27 J (safe value)

T_0 temperature (random variability of T_{27J}: μ = 18 °C, σ= 15 °C)

B plate thickness in [mm]

Table 3.1 Probabilistic models for the random variables,

Units: mm and N

Variable		**Distribution**	**Mean**	**V**	
C	Material parameter S-N curve	lognormal	$1.0 \cdot 10^{13}$	0.58	
m	Slope value		3	-	
log C_1	Material parameter 2 par S-N curve	normal	Depends		C_1 and C_2 fully correlated
log C_2	Material parameter 2 par S-N curve	normal	Depends		
m_1 (air)	Slope value 1st branch	deterministic	5	-	
m_2 (air)	Slope value 2nd branch	deterministic	3	-	
D_{cr}	Miner's sum at failure	lognormal	1.0	0.3	
A_1 (air)*	Paris Law Parameter 1	lognormal	$4.80 \cdot 10^{-18}$	1.70	
A_2 (air)*	Paris Law Parameter 2	lognormal	$5.86 \cdot 10^{-13}$	0.60	
m_1 (air)	Slope value 1st branch	deterministic	5.10	-	
m_2 (air)	Slope value 2nd branch	deterministic	2.88	-	
ΔK_0 (air)	Threshold value for ΔK	lognormal	140	0.40	
A_1 (marine)*	Paris Law Parameter 1	lognormal	$5.37 \cdot 10^{-14}$	1.10	
A_2 (marine)*	Paris Law Parameter 2	lognormal	$5.67 \cdot 10^{-7}$	0.16	
m_1 (marine)	Slope value 1st branch	deterministic	3.42	-	
m_2 (marine)	Slope value 2nd branch	deterministic	1.11	-	
ΔK_0 (marine)	Threshold value for ΔK	lognormal	0.0	-	
a_0	Initial crack depth	lognormal	0.15	0.66	
a_0/c_0	Initial aspect ratio	lognormal	0.62	0.40	
B_{glob}	MU global stress model*	lognormal	1.0	0.10	
B_{scf}	MU stress concentration	lognormal	1.0	0.20	
B_{sif}	MU stress intensity factor (hand)	lognormal	1.0	0.20	
B_{sif}	MU stress intensity factor (FEM)	lognormal	1.0	0.07	
σ_{res}	Residual stresses	lognormal	300	0.20	
R	Resistance fracture toughness	lognormal	1.7	0.18	
K_{mat}	Fracture toughness	Weibull	See (4.1)		

*stress ratio $\sigma_{min} / \sigma_{max} \geq 0.5$
**MU = Model Uncertainty

Annex A: The stress intensity correction factor *Y*

The stress intensity correction factor Y for *membrane* loading is determined with the procedure of Newman and Raju (1981) and BS 7910:2005. For the stress intensity correction factor in general they propose:

$$Y = M\, f_w M_m M_k$$

Factor M_k accounts for the local stress concentration and is provided in Annex A.2. The other factors are provided in Annex A.1. The resulting factors Ya and Yc (in height and width directions) are provided in Annex A.3

A1. The factors *M*, M_m and f_w

The equations and values provided below apply to membrane loading on flat plates with semi-elliptical cracks.
The factor M accounts for bulging effects. For flat plates, $M = 1$.
The finite-width correction function f_w is given by:

$$f_w = \left[\sec\left(\frac{\pi c}{W}\sqrt{\frac{a}{B}}\right)\right]^{1/2} \qquad \text{for } (2c/W) \le 0.8$$

The stress intensity magnification factor for semi-elliptical cracks loaded in bending is equal to:

$$M_m = \left[M_1 + M_2\left(\frac{a}{B}\right)^2 + M_3\left(\frac{a}{B}\right)^4\right] g\, f_\theta / \Phi$$

Where:

$M_1 = 1.13 - 0.09\,(a/c)$ for $0 \le a/2c \le 0.5$
$M_1 = \sqrt{(c/a)}\left[1 + 0.04(c/a)\right]$ for $0.5 < a/2c \le 1.0$
$M_2 = \dfrac{0.89}{0.2 + (a/c)} - 0.54$ for $0 \le a/2c \le 0.5$
$M_2 = 0.2\,(c/a)^4$ for $0.5 < a/2c \le 1.0$
$M_3 = 0.5 - \dfrac{1.0}{0.65 + (a/c)} + 14\left[1.0 - (a/c)\right]^{24}$ for $a/2c \le 0.5$
$M_3 = -0.11\,(c/a)^4$ for $0.5 < a/2c \le 1.0$
$g = 1 + [0.1 + 0.35\,(a/B)^2](1 - \sin\theta)^2$ for $a/2c \le 0.5$
$g = 1 + [0.1 + 0.35\,(c/a)\,(a/B)^2](1 - \sin\theta)^2$ for $0.5 < a/2c \le 1.0$

The function f_θ, an angular function from the embedded elliptical-crack solution, is:

$f_\theta = [(a/c)^2 \cos^2\theta + \sin^2\theta]^{1/4}$ for $0 \le a/2c \le 0.5$
$f_\theta = [(c/a)^2 \sin^2\theta + \cos^2\theta]^{1/4}$ for $0.5 < a/2c \le 1.0$

The complete elliptic integral of the second kind Φ is given by:

$$\Phi = \sqrt{1 + 1.464\left(\frac{a}{c}\right)^{1.65}} \qquad \text{for } 0 \le a/2c \le 0.5$$

$$\Phi = \sqrt{1 + 1.464\left(\frac{c}{a}\right)^{1.65}} \qquad \text{for } 0.5 < a/2c \le 1.0$$

The definitions of a, c and θ are provided in Fig.A1.

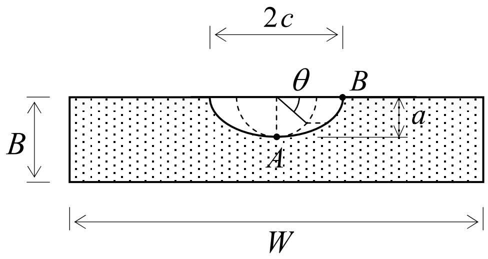


Fig A1: Semi-elliptical crack geometry near weld toe.

A2. The weld geometry factor M_k

The weld geometry M_k has to be incorporated if the crack is affected by a local stress concentration due to welding. The general formula is given by:

$$M_k = D\,(a/B)^{k1}\,(L/B)^{k2} \qquad M_k > 1$$

The parameters D and k depend on the structural and weld geometry as discribed by a, B and L. For butt welds and X-joints a set of formulas for M_{ka} values was derived by Maddox et al. (1986). These formulas are valid for weld toe angle ɵ = 45° and weld toe radius ρ = 0. The functions and the range of applicability are given in table A2.1.

Table A2.1:Stress intensity concentration factors (M_{ka}) for butt welds and X-joints
a=crack depth, B = plate thickness and L = size of the node, defined acc. to Fig. A.2.

Loading mode	**Applicability**		**Stress intensity concentration factor *)**
	L/B	*a/B*	$M_{ka} = f_L\,(a/B, L/B)$
axial	≤ 2	$\leq 0.05\,(L/B)^{0.55}$	$0.51\,(L/B)^{0.27}\,(a/B)^{-0.31}$
		$> 0.05\,(L/B)^{0.55}$	$0.83\,(a/B)^{-0.15(L/B)^{0.46}}$
	> 2	≤ 0.073	$0.615\,(a/B)^{-0.31}$
		> 0.073	$0.83\,(a/B)^{-0.20}$
bending	≤ 1	$\leq 0.03\,(L/B)^{0.55}$	$0.45\,(L/B)^{0.21}\,(a/B)^{-0.31}$
		$> 0.03\,(L/B)^{0.55}$	$0.68(a/B)^{-0.19(L/B)^{0.21}}$
	> 1	≤ 0.03	$0.45\,(a/B)^{-0.31}$
		> 0.03	$0.68\,(a/B)^{-0.19}$

For M_{kc} we may use the same formulas, however substitute a = 0,15 mm (See BS 7910:2005).

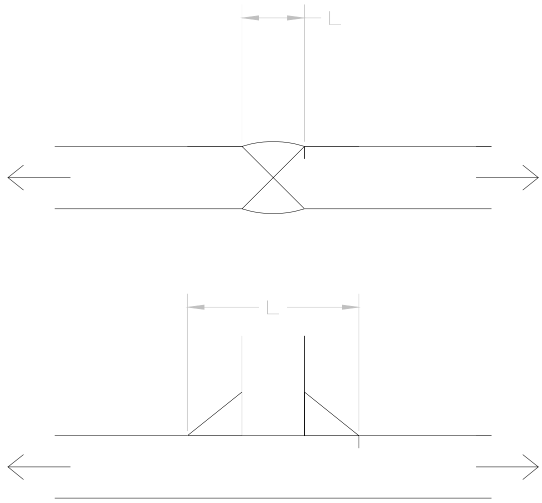


Fig A2: Definition of node length L for two weld details.

A3. The stress intensity correction factors Y_a and Y_c

For the factor Y_a it holds that $\theta = \pi/2$, therefore:

$g = 1$; $f_\theta = 1$ for $0 \le a/2c \le 0.5$ and $f_\theta = \sqrt{(c/a)}$ for $0.5 < (a/2c) \le 1.0$

For Y_c with $\theta = 0$ it follows:

$g = 1.1 + 0.35\,(a/\mathrm{B})^2$; $f_\theta = \sqrt{a/c}$ for $0 \le a/2c \le 0.5$ and $f_\theta = 1$ for $0.5 < (a/2c) \le 1.0$

Both factors finally become:

$$Y_a = M_{ka}\left[M_1 + M_2\left(\frac{a}{B}\right)^2 + M_3\left(\frac{a}{B}\right)^4\right] f_w / \Phi \qquad \text{for } 0 \le a/2c \le 0.5$$

$$Y_a = M_{ka}\left[M_1 + M_2\left(\frac{a}{B}\right)^2 + M_3\left(\frac{a}{B}\right)^4\right] f_w \sqrt{(c/a)} / \Phi \qquad \text{for } 0.5 < (a/2c) \le 1.0$$

$$Y_c = M_{kc}\left[M_1 + M_2\left(\frac{a}{B}\right)^2 + M_3\left(\frac{a}{B}\right)^4\right] f_w \left\{1.1 + 0.35\,(\mathrm{a/B})^2\right\} \sqrt{(c/a)} / \Phi \qquad \text{for } 0 \le a/2c \le 0.5$$

$$Y_c = M_{kc}\left[M_1 + M_2\left(\frac{a}{B}\right)^2 + M_3\left(\frac{a}{B}\right)^4\right] f_w \left\{1.1 + 0.35\,(\mathrm{a/B})^2\right\} / \Phi \qquad \text{for } 0.5 < (a/2c) \le 1.0$$

Appendix B: Fatigue Loading evaluation

In the S-N approach as well as in the Fracture Mechanics approach the loading term contains the expectation of the m^{th} moment of the (ergodic part) of the stress distribution.

B1 Narrow band

Rayleigh distribution

If the stress spectrum is Gaussian and narrow-banded it can be shown that the peaks of the stress process, and hence the stress ranges, follow a Rayleigh distribution. For this case the m^{th} moment of the stress range density function can be obtained as:

$$E[\Delta S^m] = (2\sigma\sqrt{2})^m \Gamma(\frac{m}{2}+1)$$

where σ is the standard deviation of the stress process and $\Gamma(\ )$ is the Gamma function.
In case we have S-N-lines or fracture mechanics models with two branches:

$$E(\Delta S^m)_0^{So} = \left(2\sigma\sqrt{2}\right)^{m1} \Gamma\left(\frac{m_1}{2}+1;\left(\frac{S_0}{2\sigma\sqrt{2}}\right)^2\right)$$

$$E(\Delta S^m)_{So}^{\infty} = \left(2\sigma\sqrt{2}\right)^{m2} \Gamma_o\left(\frac{m_2}{2}+1;\left(\frac{S_0}{2\sigma\sqrt{2}}\right)^2\right)$$

Two parameter Weibull distribution

Assume that the stress ranges are Weibull distributed with distribution function:

$$F_{\Delta s}(s) = 1 - exp\left(-\left(\frac{s}{k}\right)^{\lambda}\right)$$

In that case we have for the stress expectations

$$E(\Delta S^m) = k^m \Gamma\left(\frac{m}{\lambda}+1\right)$$

In case we have S-N-lines or fracture mechanics models with two branches:

$$E(\Delta S^m)_0^{So} = k^{m1} \Gamma\left(\frac{m_1}{\lambda}+1;\left(\frac{S_0}{k}\right)^{\lambda}\right)$$

$$E(\Delta S^m)_{So}^{\infty} = k^{m2} \Gamma_o\left(\frac{m_2}{\lambda}+1;\left(\frac{S_0}{k}\right)^{\lambda}\right)$$

Where $\Gamma(a;b)$ denotes the Incomplete Gamma function and $\Gamma_0(a;b) = \Gamma(a) - \Gamma(a;b)$ the complementary Gamma function
Note that the Rayleigh distribution for the stress ranges corresponds to a Weibull distribution with λ=2 and k = $2\sigma\sqrt{2}$, where σ is the standard deviation of the underlying Gaussian stress process.

B2 Broad banded processes:

When the stress process is broad-banded, the stress cycles cannot be easily distinguished, and a convention is required for defining stress cycles. Broadly, there are two approaches available for the modelling of stress cycles of a broad-banded stress process:

Cycle Counting Methods

The first approach is to simulate the time history of the stress process and use one of the peak, range or rain flow counting schemes to count the stress cycles, see Dowling (1972). The rain flow counting method is seen to give a good correlation with experimental results of fatigue tests under variable amplitude loading.

Semi-Analytical Probability Distributions

In the second approach, analytical distributions for stress ranges of a wide banded stress process are derived empirically, like the following 5-parameter mixed Weibull distribution by Zhao and Baker (1990):

$$f_Y(y) = \gamma \frac{b_w}{a_w}\left(\frac{y}{a_w}\right)^{b_w-1} \exp\left[-\left(\frac{y}{a_w}\right)^{b_w}\right] + (1-\gamma)\frac{d_w}{c_w}\left(\frac{y}{c_w}\right)^{d_w-1} \exp\left[-\left(\frac{y}{c_w}\right)^{d_w}\right]$$

$$\begin{aligned} a_w &= (8-7\alpha)^{-1/b} \\ b_w &= 1.1 && \text{for} \quad \alpha \le 0.9 \\ b_w &= 1.1 + 9(\alpha - 0.9) && \text{for} \quad \alpha > 0.9 \\ c_w &= \sqrt{2} \\ d_w &= 2 \end{aligned}$$

$$\alpha = \frac{m_2}{\sqrt{m_0\ m_4}}$$

$$\gamma = \frac{1-\alpha}{1-\sqrt{2/\pi}\ a_w\ \Gamma(\frac{1}{b_w}+1)}$$

where $y = \Delta S/2\sigma$ is the normalised stress range and σ is the standard deviation or ´root mean squared´ (rms) value of the stress process; m_0, m_2 and m_4 are the zero , second and fourth moment of the stress process.

For this distribution the m^{th} expected value of stress range $E[y^m]$ can be calculated:

$$E[y^m] = \gamma\ a_w^m\ \Gamma(\frac{m+b_w}{b_w}; y_{th}) + (1-\gamma)\ c_w^m\ \Gamma(\frac{m+d_w}{d_w}; y_{th}) \quad ; \quad y_{th} = \frac{S_{th}}{2\sigma}$$

where $\Gamma(\cdots;\cdots)$ is the incomplete Gamma function. When the expectation is over all stress cycles, this can be replaced by the complete Gamma function in the above expressions. In fracture mechanics S_{th} is defined in terms of the stress range corresponding to ΔK_{th} via Eq (3.5) and can be seen as a function of the random crack depth a and random aspect ratio a/c. In the case of an S-N approach, the threshold may refer to some non-propagating stress or cut off value.

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