JCSS PROBABILISTIC MODEL CODE
PART 3: RESISTANCE MODELS

BY C.-A. GRAUBNER AND E. BREHM

3.2 MASONRY PROPERTIES

List of symbols:

f_m	=	compressive strength of masonry
$f_{m,l}$	=	compressive strength of masonry in longitudinal direction
f_b	=	compressive strength of unit
f_{mo}	=	compressive strength of mortar
$f_{bt,l}$	=	tensile strength of units in longitudinal direction of the units
$f_{bt,s}$	=	splitting tensile strength of units
f_t	=	tensile strength of masonry
f_v	=	cohesion
a	=	coefficient
b	=	coefficient
c	=	coefficient
c_1	=	ratio E_m/f_m
c_2	=	ratio $f_{bt,l}/f_b$
c_3	=	ratio $f_{bt,s}/f_b$
Y_1	=	a log-normal variable taking into account uncertainties in the calculation of the compressive strength of masonry
Y_2	=	a log-normal variable taking into account uncertainties in the calculation of the modulus of elasticity
Y_3	=	a log-normal variable taking into account uncertainties in the calculation of the tensile strength of masonry
Y_4	=	a log-normal variable taking into account uncertainties in the calculation of the tensile strength of unit
Y_5	=	a log-normal variable taking into account uncertainties in the calculation of the cohesion
Y_6	=	a log-normal variable taking into account uncertainties in the calculation of the friction coefficient
ε	=	strain
ε_u	=	ultimate strain
E_0	=	modulus of elasticity
μ	=	friction coefficient
σ	=	stress
h_k	=	buckling length of wall
t	=	thickness of wall

Abbreviations:

LC	Lightweight concrete
NC	Normal density concrete
C	Concrete
CS	Calcium silicate
AAC	Autoclave aerated concrete
CB	Clay brick
GPM	General purpose mortar
TLM	Thin layer mortar

3.2.1 Scope

Masonry is made of units and mortar which show strong regional differences in the material properties and dimensions. Since stochastic treatment of masonry is relatively new, all the values given in the following represent prior values and may be subject to updating techniques. Also, the values (except the values for the compressive strength of masonry) are values derived from tests on single elements. The values might be different for full walls due to spatial variation.

This section especially deals with unreinforced masonry made of artificial units that meet the requirements of the national manufacturing codes. Masonry is widely used as an unreinforced material but in certain countries reinforced solutions are common (e.g. in earthquake regions). For reinforced masonry, additional information is required for the stochastic modelling of the reinforcing steel (s. chapter 3.03).

3.2.2 Basic Properties

The main property of masonry is the compressive strength f_m which is determined on standard test specimens (so called RILEM specimens which are 5 units tall, 2.5 units wide and 1 unit thick) tested according to standard conditions (RILEM TC 76-LUM). Other properties, such as the modulus of elasticity are related to the compressive strength. However, some properties - like the friction coefficient - directly depend on the used type of units and mortar.

Masonry only provides very small tensile strength perpendicular to the bed joints which therefore is often neglected. However, the flexural tensile strength of masonry may have a large effect on the load-carrying capacity of slender walls and should be considered in a probabilistic analysis. The flexural tensile strength parallel to the bed joints often has to be taken into account if walls are subjected to lateral loads (wind, earth pressure).

The shear capacity of masonry walls depends on the tensile strength of the units (shear failure) and on the cohesion between units and mortar (sliding failure). Additionally, the friction coefficient is influencing the tensile strength parallel to the bed joints and the sliding shear capacity.

3.2.2.1 Compressive strength of masonry

The compressive strength of masonry depends on the compressive strength of units and mortar. The following equation (1) is recommended for the determination of the compressive strength.

$$f_m = K \cdot f_b^{\alpha} \cdot f_{mo}^{\beta} \qquad (f_b \text{ and } f_m \text{ represent mean values}) \tag{1}$$

In case of thin layer mortar, the mortar quality does not influence the compressive strength of masonry and so the relationship becomes

$$f_m = K \cdot f_b^{\alpha} \qquad (f_b \text{ represents the mean value}) \tag{2}$$

with

f_m	=	mean of the compressive strength of masonry
f_b	=	mean of the compressive strength of units in vertical direction
f_{mo}	=	mean of the compressive strength of mortar
K	=	coefficient, see Table 3.2-1
α	=	coefficient, see Table 3.2-1
β	=	coefficient, see Table 3.2-1

The slenderness $\lambda = h_k/t$ plays an important role in the determination of the masonry compressive strength. To transform the obtained compressive strength to the reference slenderness of $\lambda = 5$, the following equation can be used.

$$k_{\lambda=5} = \left(0.966 + 0.00136 \cdot \lambda_{specimen}^2\right) \tag{3}$$

The parameters K, α, and β are prior values and should be updated by use of test data. The compressive strength of masonry $f_{m,l}$ in longitudinal direction (only for masonry with grouted head joints) is thought to be 50% of the masonry compressive strength f_m according to equation 1 and 2.

Table 3.2-1 Parameters K, α, β for the determination of the compressive strength of masonry according to *Schubert (2010)* for a slenderness of the specimen of $\lambda = 10$

Masonry						
Units		Mortar	*n*	*K*	*α*	*β*
Material	Unit type					
LC	full and hollow blocks	TLM	35	0.85	0.84	0
		LWM	80	0.85	0.58	0.15
		GPM	167	0.85	0.73	0.07
	full blocks	LWM	21	0.70	0.66	0.16
	hollow blocks	LWM	59	0.86	0.57	0.14
	full blocks	GPM	61	0.85	0.72	0.09
	hollow blocks	GPM	106	0.89	0.69	0.05
	full blocks	TLM	20	0.63	1.00	0
AAC	regular unit	NM	140	0.98	0.68	0.02
				0.99	0.69	0
		LM	17	0.80	0.64	0.09
				0.99	0.64	0
	plane element	DM	162	0.63	1.00	0
				0.83	0.86	0
NC	hollow block	GPM	15	0.03	1.82	0.23
CS	full	GPM	276	0.70	0.74	0.21
	block	GPM	24	0.44	0.92	0.17
	perforated	GPM	108	0.85	0.57	0.20
	hollow	GPM	70	0.99	0.64	0.05
	plane elements	TLM	66	0.53	1.00	0
CB	full	GPM	55	0.73	0.73	0.16
	perforated		342	0.55	0.56	0.46
	lightweight perforated	TLM	9	0.75	0.72	0
		LWM 21	17	0.67	0.50	0.05
		LWM 21	17	0.18	1.00	0
		LWM 36	13	0.47	0.82	0
		LWM 36	13	0.28	1.00	0
		GPM	28	0.26	0.82	0.42

3.2.2.2 Modulus of elasticity

The modulus of elasticity is commonly related to the compressive strength of masonry. For the stochastic modelling, the following equation (4) is recommended.

$$E_m = c_1 \cdot f_m \tag{4}$$

with

E_m = mean of modulus of elasticity of masonry
c_1 = ratio E_m/f_m according to Table 3.2-2

The parameter c_1 is a prior value and should be updated by use of test data.

Table 3.2-2 Values for c_1 according to Schubert (2010)

Type of unit	Mortar	c_1 [-]
CS	GPM, TLM	500
AAC	GPM	520
	TLM	560
LC	GPM	1040
	TLM	930
Perforated clay bricks	GPM	1170
	TLM	1190
	lightweight	1480

3.2.2.3 Flexural tensile strength of masonry

The flexural tensile strength of masonry can be divided into flexural tensile strength perpendicular and parallel to the bed joints. It is a strongly scattering property and depends mainly on the mortar. The execution of the head joints (unfilled (uf) or filled (f)) was also found to have small influence. Since only thin layer mortar is providing sufficiently reliable cohesion, tests of the flexural tensile strength were more or less only conducted on masonry with TLM. Values of the flexural tensile strength for masonry with TLM can be found in Table 3.2-3 and Table 3.2-4.

Table 3.2-3 Flexural tensile strength $f_{t,m,pa}$ of masonry (parallel to bed joints) according to Schubert (2010)

Unit		Head joints[a]	Number of tests n	$f_{t,m,pa}$ in N/mm²	Range of values in N/mm²
Type	Compressive strength of brick f_b in N/mm²				
Lightweight perforated CB	> 8, >12	uf	2	0.21	0.20; 0.22
CS	>12, >20	f	6	0.51	0.36÷0.69
	>28	f	2	1.05	0.96; 1.14
	>8…>28	uf	11	0.71	0.38÷0.97
Perforated CS	>12	f	4	0.48	0.45÷0.51
	>12	uf	4	0.25	0.29÷0.35
AAC	>2	f	3	0.23	0.22÷0.23
	>4…>8	f	7	0.45	0.28÷0.64
	>2, >4	uf	6	0.20	0.16÷0.24

[a]uf=unfilled, f=filled

Table 3.2-4 Flexural tensile strength of masonry $f_{t,m,per}$ (perpendicular to bed joints) according to Schubert (2010)

Unit		**Head joints**[a]	***n***	**$f_{t,m,per}$ in N/mm²**	**Range of values in N/mm²**
Type	**Compressive strength of brick f_b in N/mm²**				
Lightweight perforated CB	> 8, >12	uf	3	0.28	0.26÷0.30
CS	-	uf/f	8	0.56	0.35÷0.73
Perforated CS	-	f	4	0.34	0.23÷0.48
AAC	>2…>8	uf/f	23	0.40	0.25÷0.81
Concrete blocks	>2…>6	f	5	0.33	0.22÷0.44
[a]uf=unfilled, f=filled					

3.2.2.4 Tensile strength of the units

For the tensile strength in longitudinal direction of the units:

$$f_{bt,l} = c_2 \cdot f_b \tag{5}$$

For the splitting tensile strength of the units:

$$f_{bt,s} = c_3 \cdot f_b \tag{6}$$

with

$f_{bt,l}$ = mean of tensile strength of the unit in longitudinal direction
$f_{bt,s}$ = mean of splitting tensile strength of the units
c_2 = ratio $f_{bt,l}/f_b$
c_3 = ratio $f_{bt,s}/f_b$

For the large variety of masonry products which all differ in perforation and material, the prior values given in Table 3.2-5 can be used.

Table 3.2-5 Values c_2 and c_3 according to *Schubert (2010)*

Type of unit	**Number of tests (c_2)**	**c_2 [-]**	**Number of tests (c_3)**	**c_3 [-]**
CS	18	0.063	40	0.070
Perforated CS	19	0.035	31	0.060
CB	9	0.040	9	0.070
Perforated CB	20	0.030	29	0.040
Lightweight perforated CB	54	0.010	-	-
Lightweight concrete hollow block	8	0.080	10	0.090
Full lightweight concrete block	23	0.080	13	0.110
AAC, plane element	24	0.110	24	0.090
AAC, plane element 2	7	0.180	9	0.150
AAC, plane element 4,6,8	8	0.110	9	0.120
Hollow Concrete block	2	0.080	3	0.040

3.2.2.5 Cohesion

The values $f_{v,m}$ for the cohesion can be taken from Table 3.2-6. Please note that these values have been derived from the characteristic values provided in EN 1996-1-1 by application of the stochastic models presented in Table 3.2-13 and assuming the characteristic values to represent 5%-quantiles. The values for the cohesion are prior values and require updating by use of test data.

Table 3.2-6 Mean of the cohesion $f_{v,m}$

$f_{v,m}$ in MN/m²			
GPM		TLM	LWM
20 and higher	0.42	0.65[a]	0.26
10 ÷ 19	0.37		
5 ÷ 9	0.28		
2.5 ÷ 4.5	0.14		
1 ÷ 2	0.04		
[a]divide by 2 if the perforation is more than 15% of the cross-section			

3.2.2.6 Friction coefficient

The friction coefficient μ is normally estimated with a mean value of 0.8. In case of in-plane shear a reduction may become necessary to account for possible overturning of the units.

3.2.3 Stress-strain relationship

In unified form, the stress-strain relationship of masonry under compression is determined by:

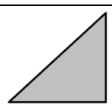
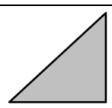
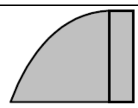
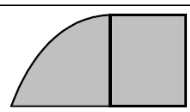
$$\frac{\sigma}{f_m} = \frac{k_0 \cdot \eta - \eta^2}{1 + (k_0 - 2) \cdot \eta} \qquad (7)$$

$$\eta = \frac{\varepsilon}{\varepsilon_f} \qquad (8)$$

$$k_0 = \frac{E_0 \cdot \varepsilon_f}{f_m} \qquad (9)$$

The value k_0 represents the slope of the unified stress-strain curve at origin. Values for the different materials can be taken from Table 3.2-7.

Table 3.2-7 Parameters for the stress-strain relationship of different kinds of masonry under compression

Lightweight concrete	Hollow clay bricks and AAC	CS hollow block	CS full block
			
k_0 = 1.0	k_0 = 1.0	k_0 = 2.0	k_0 = 2.0
ε_f = 1.2 ‰	ε_f = 2.0 ‰	ε_f = 2.0 ‰	ε_f = 2.0 ‰
ε_u = 1.2 ‰	ε_u = 2.0 ‰	ε_u = 2.5 ‰	ε_u = 3.5 ‰

3.2.4 The probabilistic model

3.2.4.1 Compressive strength

The compressive strength of masonry depends on the compressive strength of units and mortar. The following equation (10) is recommended for the determination of the compressive strength.

$$f_{m,j} = Y_1 \cdot f_m \tag{10}$$

with

f_m = compressive strength of masonry according to equation (1) or (2)
Y_1 = a log-normal variable accounting for uncertainties in the calculation of the compressive strength, see Table 3.2-8

Table 3.2-8 Stochastic Parameters for Y_1

Unit	Mortar	Distr.	Mean	CoV
CS	TLM	LN	1.0	20%
AAC				20%
Large sized units CS				16%
Large sized units AAC				14%
Clay bricks	GPM			17%

3.2.4.2 Modulus of elasticity

The modulus of elasticity is commonly related to the compressive strength of masonry. For the stochastic modelling, the following equation (11) is recommended. A first estimate for the stochastic parameters of the log-normal variable Y_2 can be found in Table 3.2.7.

$$E_{m,j} = Y_2 \cdot E_m \tag{11}$$

with

E_m = modulus of elasticity according to equation (4)
Y_2 = a log-normal variable

Table 3.2-9 Stochastic Parameters for Y_2

Unit	Mortar	Distr.	Mean	CoV
all	all types	LN	1.0	25%

3.2.4.3 Flexural tensile strength of masonry

There is significant lack of data for the tensile strength of masonry. Nevertheless, the values in Table 3.2.3 may be taken as first estimates for the means. The flexural tensile strength of masonry may be modelled as follows:

$$f_{t,j} = Y_3 \cdot f_{t,m,i} \tag{12}$$

with

$f_{t,m}$ = mean of flexural tensile strength according to Table 3.2-3 and Table 3.2-4
Y_3 = a log-normal variable

The stochastic parameters for the log-normal variable Y_3 can be found in Table 3.2-10 depending on the direction of the flexural tensile strength (parallel or perpendicular to the bed joint). Please note, these values only represent prior parameters and require updating and validation by use of test data.

Table 3.2-10 Stochastic Parameters for Y_3

Unit	Mortar	Direction	Distr.	mean	CoV
CS	TLM	parallel	LN	1.0	30%
		perpen.			35%
AAC	TLM	parallel			26%
		perpen.			30%
Lightweight perforated CB	TLM	parallel			30%
		perpen.			35%
Concrete blocks	TLM	perpen.			30%

3.2.4.4 Tensile strength of the units

The shear capacity of masonry walls is strongly influenced by the tensile strength of units. As for many materials, the tensile strength can be described as function of the compressive strength. It was found that the tensile strength in longitudinal direction describes the shear capacity of CS and AAC units well while in case of CB, the splitting tensile strength is preferable.

For the tensile strength in longitudinal direction of the units of CB and AAC units:

$$f_{bt,l,j} = Y_4 \cdot f_{bt,l} \quad (13)$$

For the splitting tensile strength of CB units:

$$f_{bt,s,j} = Y_4 \cdot f_{bt,s} \quad (14)$$

with

$f_{bt,l}$ = the tensile strength of the units in the longitudinal direction according to equation (5)
$f_{bt,s}$ = the splitting tensile strength of the units according to equation (6)
Y_4 = a log-normal variable accounting for the large scatter of the tensile strength of the units. Values can be found in Table 3.2-11.

Table 3.2-11 Stochastic Parameters for Y_4

Unit	Distr.	Mean	CoV
CS	LN	1.0	26%
AAC			16%
Clay bricks			24%

3.2.4.5 Cohesion

The cohesion may be modelled according to equation (15). A first estimate for the stochastic parameters for the random variable Y_5 may be taken from Table 3.2.9 for different combinations of units and mortar.

$$f_{v,j} = Y_5 \cdot f_{v,m} \quad (15)$$

with

$f_{v,m}$ = mean of the cohesion according to Table 3.2-6
Y_5 = a log-normal variable according to Table 3.2-12

Table 3.2-12 Stochastic Parameters for Y_5

Unit	Mortar	Dist.	Mean	CoV
AAC	TLM	LN	1.0	35%
CS				
CB	IIa			40%

3.2.4.6 Friction coefficient

The friction coefficient may be modelled according to equation (16) by use of the stochastic parameters found in Table 3.2-11.

$$\mu_j = Y_6 \cdot \mu_m \tag{16}$$

with

μ_m = 0.8
Y_6 = a log-normal variable according to Table 3.2-13

Table 3.2-13 **Stochastic Parameters for** Y_6

Unit	Distr.	Mean	CoV
CS	LN	1.0	19%
AAC			

References

Glowienka, S. (2007): Zuverlässigkeit von Mauerwerkswänden aus großformatigen Steinen, doctoral thesis, Technische Universität Darmstadt, Darmstadt, in German

Schueremans, L. (2001): Probabilistic Evaluation of structural masonry, doctoral thesis, University of Leuven, Belgium, 2001

Glock, C. (2004): Traglast unbewehrter Beton- und Mauerwerkswände, doctoral thesis, Technische Universität Darmstadt, Darmstadt, in German

Schubert, P. (2010): Eigenschaftswerte von Mauerwerk, Mauersteinen und Mauermörtel, Mauerwerk-Kalender 2010, Ernst & Sohn, Berlin

EN 1996-1-1 (Eurocode 6), Design of masonry structures, Beuth Verlag, Berlin, 2005, AC: 2009

Brehm, E. (2011): Reliability of Unreinforced Masonry Bracing Walls, doctoral thesis, ISBN 978-3-942886-02-4, Technische Universität Darmstadt