Reliability Based Code Calibration

Joint Committee on Structural Safety

M.H. Faber
Swiss Federal Institute of Technology, Zürich, Switzerland.

J.D. Sørensen
Aalborg University, Aalborg, Denmark.

Keywords: Code calibration, stochastic models, target reliabilities, partial safety factors, reliability analysis, decision theory

Summary
The present paper addresses fundamental concepts of reliability based code calibration. First basic principles of structural reliability theory are introduced and it is shown how the results of FORM based reliability analysis may be related to the partial safety factors and characteristic values. Thereafter the code calibration problem is presented in its principal decision theoretical form and it is discussed how acceptable levels of failure probability (or target reliabilities) may be established. Furthermore suggested values for acceptable annual failure probabilities are given for the ultimate and the serviceability limit states. Finally the paper describes a procedure for the practical implementation of reliability based code calibration of LRFD based design codes. In conclusion a number of important aspects are discussed relating to the difficulties encountered in practice when design codes are calibrated across a broad variety of types of structures, materials and loading conditions.

1. Introduction
Ultimately structural design codes are established for the purpose of providing a simple, safe and economically efficient basis for the design of ordinary structures under normal loading, operational and environmental conditions. Design codes thereby not only greatly facilitate the daily work of structural engineers but also provide the vehicle to ensure a certain standardization within the structural engineering profession which in the end enhances an optimal use of the resources of society for the benefit of the individual.

Traditionally design codes take basis in design equations from which the reliability verification of a given design may be easily performed by a simple comparison of resistances and loads and/or load effects. Due to the fact that loads and resistances are subject to uncertainties, design values for resistances and load effects are introduced in the design equations to ensure that the design is associated with an adequate level of reliability. Design values for resistances are introduced as a characteristic value of the resistance divided by a partial safety factor (typically larger than 1) and design values for load effects are introduced as characteristic values multiplied by a partial safety factor (typically larger than 1). Furthermore in order to take into account the effect of simultaneously occurring variable load effects so-called load combination factors (smaller than 1) are multiplied on one or more of the variable loads.
Over the years different approaches for establishing design values for resistances and loads have been applied in different countries. Within the last decade, however, almost all design codes have adopted the Load and Resistance Factor Design format (LRFD). Different versions of the LRFD format exist see e.g. CIRIA [1], CEB [2] and [3], Eurocodes [4], AHISTO [5] and OHBDC [6], but they are essentially based on the same principles.

The structural engineering profession has an exceptionally long tradition going several thousand years back. During these years experience and expertise has been collected to some extent by trial and error. The design of new types of structures, with new materials or subject to new loading and environmental conditions had to be performed in an adaptive manner based on careful and/or “conservative” extrapolations of existing experience. The results were not always satisfactorily and some iteration has in general been necessary. In fact one may consider the present structural engineering traditions as being the accumulated experience and knowledge collected over this long period. This applies not least to the level of inherent safety with which the present engineering structures are being designed.

The development of structural reliability methods during the last 3 to 4 decades have provided a more rational basis for the design of structures in the sense that these methods facilitate a consistent basis for comparison between the reliability of well tested structural design and the reliability of new types of structures. For this reason the methods of structural reliability have been applied increasingly in connection with the development of new design codes over the last decades.

By means of structural reliability methods the safety formats of the design codes i.e. the design equations, characteristic values and partial safety factors may be chosen such that the level of reliability of all structures designed according to the design codes is homogeneous and independent of the choice of material and the prevailing loading, operational and environmental conditions. This process including the choice of the desired level of reliability or “target reliability” is commonly understood as “code calibration”. Reliability based code calibration has been formulated by several researchers, see e.g. Ravindra and Galambos [7], Ellingwood et al. [8] and Rosenblueth and Esteva [9] and has also been implemented in several codes, see e.g. OHBDC [6], NBCC [10] and more resent the Eurocodes [4].

The present paper attempts to give an overview of the methodology applied in reliability based code calibration. First a short description of the LRFD safety format is given. Secondly the fundamental principles of structural reliability theory and the relation between reliability analysis results and the LRFD safety format are explained. Thereafter a decision theoretical formulation of the code calibration problem is formulated, the issue concerning the choice of target reliabilities is discussed and guidelines are given for the rational treatment of this problem. Finally a practically applicable approach for reliability based code calibration is proposed.

2. Basic principles of structural reliability

The overall aim of structural reliability analysis is to quantify the reliability of structures under consideration of the uncertainties associated with the resistances and loads. The structural performance is assessed by means of models based on physical understanding and empirical data. Due to idealizations, inherent physical uncertainties and inadequate or insufficient data the models themselves and the parameters entering the models such as material parameters and load characteristics are uncertain. Structural reliability theory takes basis in the probabilistic model-
ing of these uncertainties and provides methods for the quantification of the probability that the structures do not fulfil the performance criteria.

2.1 Uncertainty modeling

The uncertainties, which must be considered, are the physical uncertainty, the statistical uncertainty and the model uncertainty. The physical uncertainties are typically uncertainties associated with the loading environment, the geometry of the structure and the material properties. The statistical uncertainties arise due to incomplete statistical information e.g. due to a small number of materials tests. Finally, the model uncertainties must be considered to take into account the uncertainty associated with the idealized mathematical descriptions used to approximate the actual physical behavior of the structure. The probabilistic modeling of uncertainties highly rests on a Bayesian statistical interpretation of uncertainties implying that the uncertainty modeling utilizes and facilitates both the incorporation of statistical evidence about uncertain parameters and subjectively assessed uncertainties. Modern methods of reliability and risk analysis allow for a very general representation of these uncertainties ranging from non-stationary stochastic processes and fields to time-invariant random variables, see e.g. Melchers [11]. In most cases it is sufficient to model the uncertain quantities by random variables with given distribution functions and distribution parameters estimated on basis of statistical and/or subjective information. In the probabilistic model code by JCSS [12] an almost complete set of probabilistic models are given covering most situations encountered in practical engineering problems.

2.2 Structural reliability and probability of failure

The performance criteria for structural systems and components are normally expressed in terms of limit state equations \( g(x) \) and so-called failure events \( F \)

\[
F = \{ g(x) \leq 0 \} \tag{1}
\]

where the components of the vector \( x \) are realizations of the so-called basic random variables \( X \) representing all the relevant uncertainties influencing the probability of failure. The basic random variables must be able to represent all types of uncertainties that are included in the analysis.

Having established probabilistic models for the uncertain variables the problem remains to evaluate the probability of failure corresponding to a specified reference period. However, also other non-failure states of the considered component or system may be of interest, such as excessive damage, unavailability, etc. In general any state, which may be associated with consequences in terms of costs, loss of lives and impact to the environment are of interest. In the following, however, for simplicity these states are not differentiated.

For a structural component for which the uncertain resistance \( R \) is modeled by a random variable with probability density function \( f_R(r) \) subjected to the load \( s \) the limit state function is simply

\[
g(x) = R - s \tag{2}
\]

and the probability of failure may be determined by

\[
P_F = P(R \leq s) = F_R(s) = P(R / s \leq 1) \tag{3}
\]
In case also the load $s$ is uncertain and modeled by the random variable $S$ with probability density function $f_S(s)$ the probability of failure is

$$P_F = P(R \leq S) = P(R - S \leq 0) = \int_{-\infty}^{\infty} F_R(x) f_S(x) dx$$

(4)

assuming that the load and the resistance variables are statistically independent. This case is called the fundamental case. The integration in (4) is illustrated in Figure 1. The contributions to the probability integral in (4) are illustrated. Note that the probability of failure is not determined through the overlap of the two curves.

There exists no general closed form solution to the integral in (4) but for a number of special cases solutions may be derived. One case is when both the resistance variable $R$ and the load variable $S$ are normally distributed. In this case the failure probability may be assessed directly by considering the random variable $M$ often referred to as the safety margin

$$M = R - S$$

(5)

whereby the probability of failure may be assessed through

$$P_F = P(R - S \leq 0) = P(M \leq 0)$$

(6)

where $M$ is also being normal distributed with parameters $\mu_M = \mu_R - \mu_S$ and standard deviation $\sigma_M = \sqrt{\sigma_R^2 + \sigma_S^2}$.

The failure probability may now be determined by use of the standard normal distribution function as

$$P_F = \Phi\left(\frac{0 - \mu_M}{\sigma_M}\right) = \Phi(-\beta)$$

(7)

where $\mu_M / \sigma_M = \beta$ is called the safety index. The geometrical interpretation of the safety index is illustrated in Figure 2.
Figure 2 Illustration of the probability density function for the normally distributed safety margin $M$.

From Figure 2 it is seen that the safety index $\beta$ may be interpreted as the number of standard deviation by which the mean value of the safety margin $M$ exceeds zero, or equivalently the distance from the mean value of the safety margin to the most likely failure point.

As indicated previously closed form solutions may also be obtained for other special cases. However, as numerical methods have been developed for the purpose of solving (4) we will not consider these in the following.

In the general case the resistance and the load cannot be described by only two random variables but rather by functions of random variables. Then the more failure probability may be assessed through the following integral

$$P_{f} = \int_{g(x) \leq 0} f_{X}(x)dx$$  \hspace{1cm} (8)

where $f_{X}(x)$ is the joint probability density function of the random variables $X = (X_1, X_2, \ldots, X_n)^T$. This integral, illustrated in Figure 3 as a volume integral of the joint density function in the failure domain is, however, non-trivial to solve and numerical approximations are expedient. Various methods for the solution of the integral in (8) have been proposed including numerical integration techniques, Monte Carlo simulation and First and Second Order Reliability Methods (FORM/SORM), see e.g. Madsen et al. [13]. Numerical integration techniques very rapidly become inefficient for increasing dimension of the vector $X$ and are in general irrelevant.

Figure 3 Illustration of the failure probability integration problem in two dimensions.
The first developments of First and Second Order Reliability Methods (FORM/SORM) took place almost 30 years ago with pioneering work performed by Basler [14], Cornell [15] and Hasofer & Lind [16]. Since then these methods together with advanced Monte Carlo simulation techniques have been refined and extended significantly and by now they form the most important methods for reliability evaluations in structural reliability theory. For the most common practical purposes the problem of estimating probabilities may be considered as solved. Several commercial computer codes have been developed for FORM/SORM and simulation analysis and the methods are widely used in practical engineering problems and not least for code calibration purposes, see e.g. STRUREL [17] and Proban [18].

2.3 Time variant reliability

In many engineering applications it is necessary to determine the reliability of structural components subject to stochastic process loading or time-varying strength/degradation. Then the probability that the structural component enters the failure region during some given time must be determined. Let failure occur when the process \( g(x(t)) \leq 0 \) for some time, \( t \) during the time interval, \( [0,T] \) considered. \( x(t) \) is a realization of the stochastic process \( \{X(t)\} \). The probability of failure in the interval \([0;T]\) is

\[
P_f(T) = 1 - P(g(X(t)) > 0, \forall t \in [0;T])
\]  

(9)

Evaluation of (9) is generally very difficult, and approximations are used in practical applications. Often an upper bound of the probability of failure in the time interval \([0,T]\) is used:

\[
P_f(T) \leq \int_0^T \nu^+(t, \xi) dt
\]  

(10)

where the out-crossing rate \( \nu^+(t, \xi) \) is determined by a suitable application of Rice’s formula, see e.g. Madsen et al. [13].

2.4 Reliability and partial safety factors

In code based design formats such as the Eurocodes [4], design equations are prescribed for the verification of the capacity of different types of structural components in regard to different modes of failure. The typical format for the verification of a structural component is given as design equations such as

\[
G = z R_c / \gamma_m - \{ \gamma_G C_c + \gamma_Q Q_c \} = 0
\]  

(11)

where
- \( R_c \) is the characteristic value for the resistance
- \( z \) is a design variable (e.g. the cross sectional area of the steel rod considered previously)
- \( C_c \) is a characteristic value for the permanent load
- \( Q_c \) is a characteristic value for the variable load
- \( \gamma_m \) is the partial safety factor for the resistance
- \( \gamma_G \) is the partial safety factor for the permanent load
- \( \gamma_Q \) is the partial safety factor for the variable load
In the codes different partial safety factors are specified for different materials and for different types of loads. Furthermore when more than one variable load is acting load combination factors are multiplied on one or more of the variable load components to take into account the fact that it is unlikely that all variable loads are acting with extreme values at the same time.

The partial safety factors together with the characteristic values are introduced in order to ensure a certain minimum reliability level for the structural components designed according to the code. As different materials have different uncertainties associated with their material parameters the partial safety factors are in general different for the different materials. The principle is illustrated in Figure 4 for the simple r-s case.

\[
f_r(r), f_s(s)
\]

\[
\gamma_Q = \frac{s_d}{s_c}, \quad \gamma_m = \frac{r_c}{r_d}
\]

Figure 4 Illustration of the relation between design values, characteristic values and partial safety factors.

In accordance with a given design equation such as e.g. (9) a reliability analysis may be made with a limit state function of the same form as the design equation but where the characteristic values for the resistance and load variables are now replaced by basic random variables, i.e.

\[
g = zR - (G + Q) = 0
\]  

(12)

For given probabilistic models for the basis random variables \( R, G \) and \( Q \) and with a given requirement to the maximum allowable failure probability it is now possible to determine the value of the design variable \( z \) which corresponds to this failure probability. Such a design could be interpreted as being an optimal design because it exactly fulfils the given requirements to structural reliability.

Having determined the optimal design \( z \) we may also calculate the corresponding design point in the original space, i.e. \( x_d \) for the basic random variables. This point may be interpreted as the most likely failure point, i.e. the most likely combination of the outcomes of the basic random variables leading to failure. Now partial safety factors may be derived from the design point for the various resistance variables as
\[ \gamma_m = \frac{x_c}{x_d} \]  

and for load variables

\[ \gamma_l = \frac{x_l}{x_c} \]

where \( x_d \) is the design point for the considered design variable and \( x_c \) the corresponding characteristic value.

For time-variant reliability problems a similar procedure can be used to determine partial safety factors.

### 3. The code calibration decision problem

In this section it is described how the code calibration problem can be formulated as a decision problem. Two levels of code calibration can be formulated, namely calibration of target reliabilities (or probabilities of failure) and direct calibration of the partial safety factors. Calibration / determination of target reliabilities are considered in section 4. Here it is described how partial safety factors using a decision theoretical approach can be calibrated. A general formulation based on decision theoretical concepts is obtained when the total expected cost-benefits for a given class of structures are maximized with the partial safety factors as decision variables, see e.g. Sørensen et al. [16]:

\[
\max_{\gamma} W(\gamma) = \sum_{j=1}^{L} w_j \left[ B_j - C_{ij}(\gamma) - C_{ij} - C_{ij} P_{ij}(\gamma) \right]
\]

s.t. \( \gamma^l_i \leq \gamma_i \leq \gamma^u_i \), \( i = 1, \ldots, m \)

where \( \gamma = (\gamma_1, \ldots, \gamma_m) \) are the \( m \) partial safety factors to be calibrated. Load combination factors will in general also be calibrated/optimized, therefore \( \gamma = (\gamma_1, \ldots, \gamma_m) \) can e assumed also to contain those load combination factors to be calibrated. \( \gamma^l_i, \gamma^u_i \) and \( \gamma^l_j, \gamma^u_j \) are lower and upper bounds on the partial safety factors. \( L \) is the number of different failure modes / limit states used to cover the application area considered. \( w_j \) is a factor indicating the relative frequency of failure mode \( j \). \( B_j \) is the expected benefits (in general for the society, but in some cases the benefits can be related to the owner of the structures considered), \( C_{ij} \) is the initial (or construction) costs, \( C_{ij} \) is the repair/maintenance costs during the design life time and \( C_{ij} \) is the cost of failure. \( C_{ij} \) is assumed to be independent of the partial safety factors. \( P_{ij} \) is the probability of failure for failure mode \( j \) if the structure is designed using given partial safety factors.

The formulation in (15) is based on single failure modes and corresponds to the single failure mode checking format used in structural codes of practice. A similar systems approach can be formulated where the probability of failure of the system can be determined assuming system failure if one of the single failure modes fails (series system model) and where systems related costs are introduced. However, the corresponding deterministic systems reliability measures (ro-
bustness measures) are difficult to identify and are generally not used in structural codes. In the following the single failure mode checking format is assumed to be used.

The limit state functions related to the failure modes considered are written:

\[ g_j(x, p_j, z) = 0 \]  

(16)

where \( p_j \) is a vector with deterministic parameters and \( z = (z_1, ..., z_N) \) are the design variables. The application area for the code is described by the set \( I \) of \( L \) different vectors \( p_j, j = 1, ..., L \). The set \( I \) may e.g. contain different geometrical forms of the structure, different parameters for the stochastic variables and different statistical models for the stochastic variables.

The deterministic design equation related to the limit state equation in (16) is written

\[ G_j(x_c, p_j, z, \gamma) \geq 0 \]  

(17)

where \( x_c = (x_{c1}, ..., x_{cn}) \) are characteristic values and \( \gamma = (\gamma_1, ..., \gamma_m) \) are the partial safety factors.

\( C_{ij}(\gamma), C_{Rj}(\gamma) \) and \( P_{Fj}(\gamma) \) can be determined on the basis of the solution of the following deterministic optimization problem where the optimal design \( z \) is determined using the design equations and given partial safety factors:

\[ \min_{\gamma} C_{ij}(z) \]

\[ s.t. \ G_j(x_c, p_j, z, \gamma) \geq 0 \]

\[ z_i^l \leq z_i \leq z_i^u, i = 1, ..., N \]  

(18)

The objective function in (18) is the construction costs, and the constraints are related to the design equations. Using the limit state equation in (16) the probability of failure of the structure \( P_{Fj} \) and the expected repair/maintenance costs \( C_{Rj} \) to be used in (15) are determined at the optimum design point \( z^* \). In cases where more than one failure mode is used to design a structure included in the code calibration, the relevant design equations all have to be satisfied for the optimal design \( z^* \). The objective function in (18) can be extended also to include the repair/maintenance costs and the benefits.

It is noted that when the partial safety factors are determined from (15) they will in general not be independent. In the simplest case with only a resistance partial safety factor and a load partial safety factor only the product of the two partial safety factors is determined.

4. Optimality and target reliabilities

It is well known, but not always fully appreciated, that the reliability of a structure as estimated on the basis of a given set of probabilistic models for loads and resistances may have limited bearing to the actual reliability of the structure. This is the case when the probabilistic modeling forming the basis of the reliability analysis is highly influenced by subjectivity and then the estimated reliability should be interpreted as being a measure for comparison only. In these cases it
is thus not immediately possible to judge whether the estimated reliability is sufficiently high without first establishing a more formalized reference for comparison.

Such a reference may be established by the definition of an optimal or best practice structure. The idea behind the "best practice" reference is that if the structure of consideration has been designed according to the "best practice" then the reliability of the structure is "optimal" according to agreed conventions for the target reliability. Typical values for the corresponding target annual failure probability are in the range of $10^{-6}$ to $10^{-7}$ depending on the type of structure and the characteristics of the considered failure mode. Using this approach the target reliability is determined as the reliability of the "best practice" design as assessed with the given probabilistic model.

The determination of the "best practice" design can be performed in different ways. The simplest approach is to use the existing codes of practice for design as a basis for the identification of "best practice" design. Alternatively the "best practice design" may be determined by consultation of a panel of recognized experts.

In case where the probabilistic modeling does not rest on subjective assessments the most rational approach is to establish the optimal design on the basis of the economic decision theory. By considering the expected total benefit $E[B]$ associated with the considered structure

$$E[B] = I - (1 - P_F(C_D)) - C_D - C_F \cdot P_F(C_D) = I - C_D - (I + C_F)P_F(C_D)$$  \hspace{1cm} (19)

where $I$ is the expected benefit from the structure, $C_F$ is the cost consequence in case of failure, $C_D$ is the cost of some risk reducing measure, e.g. an increase of a dimension, and where the probability of failure is a function of the costs invested in the risk reduction we have that the optimal investment in risk reducing measures may be determined from the following optimality criterion.

$$\frac{\partial E[B]}{\partial C_D} = -1 - (I + C_F) \cdot \frac{\partial P_F(C_D)}{\partial C_D} = 0$$  \hspace{1cm} (20)

from which the cost efficient level of risk reducing measures may be determined. Having determined these we may, by application of (17) assess the feasibility of the considered structure by recognizing that the total expected benefit of the structure shall be larger than zero.

Without going in to the details prevailing the derivations it is interesting to notice that it is possible, based on recent research work by Nathwani and Lind [20] and Rackwitz [21] to establish optimal values for risk reduction costs when also the consequences of loss of human lives are considered by means of the Life Quality Index. The Life Quality Index, $L$ is a compound social indicator defined as

$$L = g^w e^{1-w}$$  \hspace{1cm} (21)

where $g$ is the gross domestic product per year per person, $e$ is the life expectancy at birth and $w$ is the proportion of life spent in economic activity. In developed countries it may be assumed that $w=1/8$. $g$ lies in the interval of $\n$
countries, 67 years in medium developed countries and 73 years in highly developed countries, see e.g. Skjong and Ronold [22]. The LQI implies that a risk reducing measure is feasible if

\[
\frac{\Delta e}{e} \geq \frac{\Delta g}{g} \frac{w}{1-w}
\]  

(22)

which may be obtained from (21) as explained in Nathwani and Lind [20]. From (22) the optimal risk reducing measure for saving the life of a person may be identified by considering the case of equality. Then we obtain

\[
|\Delta g|_{\text{max}} = \frac{g}{e} \frac{1-w}{w} \Delta e = \frac{g}{2} \frac{1-w}{w}
\]  

(23)

which may be interpreted as the optimal acceptable costs per life year saved and where it has been assumed that number of life years saved by saving one individual \( \Delta e \) in average equals \( \Delta e = \frac{e}{2} \).

From (21) we may now readily calculate to optimal costs of saving the life of one individual, also called the optimum acceptable implied cost of averting a fatality (ICAF) from

\[
\text{ICAF} = \frac{ge}{4} \frac{1-w}{w}
\]  

(24)

from which it may be found that optimum values of ICAF lies in the range of $US 2 – 3 x 10^6$.

These costs may be included in (19) when the optimal investments into safety are considered and thus treated within the same framework as any other asset loss. It should be noticed that as a consequence hereof the acceptable failure probability associated with a specific project or structure depends on its specific characteristic, i.e. the monetary consequences in case of failure together with the expected benefits of the activity.

In Tables 1 - 2 target failure probabilities and corresponding target reliability indexes are given for ultimate limit states and serviceability limit states, respectively based on the recommendations of JCSS [12]. Note that the values given correspond to a year reference period and the stochastic models recommended in JCSS [12].

**Table 1: Tentative target reliability indices \( \beta \) (and associated target failure probabilities) related to a one-year reference period and ultimate limit states**

<table>
<thead>
<tr>
<th>Relative cost of safety measure</th>
<th>Minor consequences of failure</th>
<th>Moderate consequences of failure</th>
<th>Large consequences of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>( \beta=3.1 ) ( (P_f \approx 10^{-3}) )</td>
<td>( \beta=3.3 ) ( (P_f \approx 5 \times 10^{-4}) )</td>
<td>( \beta=3.7 ) ( (P_f \approx 10^{-4}) )</td>
</tr>
<tr>
<td>Normal</td>
<td>( \beta=3.7 ) ( (P_f \approx 10^{-3}) )</td>
<td>( \beta=4.2 ) ( (P_f \approx 10^{-5}) )</td>
<td>( \beta=4.4 ) ( (P_f \approx 5 \times 10^{-5}) )</td>
</tr>
<tr>
<td>Low</td>
<td>( \beta=4.2 ) ( (P_f \approx 10^{-5}) )</td>
<td>( \beta=4.4 ) ( (P_f \approx 10^{-5}) )</td>
<td>( \beta=4.7 ) ( (P_f \approx 10^{-6}) )</td>
</tr>
</tbody>
</table>

**Table 2: Tentative target reliability indices (and associated probabilities) related to a one-year reference period and irreversible serviceability limit states**
<table>
<thead>
<tr>
<th>Relative cost of safety measure</th>
<th>Target index (irreversible SLS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>$\beta = 1.3 \ (P_F \approx 10^{-1})$</td>
</tr>
<tr>
<td>Normal</td>
<td>$\beta = 1.7 \ (P_F \approx 5 \times 10^{-2})$</td>
</tr>
<tr>
<td>Low</td>
<td>$\beta = 2.3 \ (P_F \approx 10^{-2})$</td>
</tr>
</tbody>
</table>

5. A practical code calibration procedure

Code calibration can be performed by judgement, fitting, optimization or a combination of these, see Madsen et al. [13] and Thoft-Christensen & Baker [23]. Calibration by judgement has been the main method until 10-20 years ago. Fitting of partial safety factors in codes is used when a new code format is introduced and the parameters in this code are determined e.g. such that the same level of safety is obtained as in the old code or calibrated to a target reliability level. In practical code optimization the following steps are generally performed:

1. **Definition of the scope of the code**
2. **Definition of the code objective**
3. **Definition of code format**
4. **Identification of typical failure modes and of stochastic model**
5. **Definition of a measure of closeness**
6. **Determination of the optimal partial safety factors for the chosen code format**
7. **Verification**

**Ad 1.**
The class of structures and the type of relevant failure modes to be considered are defined.

**Ad 2.**
The code objective may be defined using target reliability indices or target probability of failures. Those can for example be based on the target reliabilities indicated in recommendations (e.g. Eurocodes [4] and ISO [24]), see section 4 or on reliabilities obtained by reliability analyses of structures designed by old, well-proven and accepted structural codes of practice. It is important to note that the target reliabilities are linked closely to the stochastic models used for the uncertain variables and the applied limit states.

**Ad 3.**
The code format includes:
- how many partial safety factors and load combination factors to be used
- should load partial safety factors be material independent, and
- should material partial safety factors be load type independent
- how to use the partial safety factors in the design equations
- rules for load combinations

In general for practical use the partial safety factors should be as few and general as possible. On the other hand a large number of partial safety factors is needed to obtain economically and safe structures for a wide range of different types of structures.

**Ad 4.**
Within the class of structures considered typical failure modes are identified. Limit state equations and design equations are formulated and stochastic models for the parameters in the limit
state equations are selected. Also the frequency at which each type of safety check is performed is determined.

The stochastic model for the uncertain parameters should be selected very carefully. Guidelines for the selection can be found in JCSS [12]. Also in the Eurocodes [4] and ISO [24] some guidelines can be found. In general the following main recommendations can be made.

Strength / resistance parameters are often modeled by Lognormal distributions. This avoids the possibility of negative realizations. In some cases it can be relevant also to consider a Weibull distribution for a material parameter. This is especially the case if the strength is governed by brittleness, size effects and material defects. The coefficient of variation varies with the material type considered. Typical values are 5 % for steel and reinforcement, 15 % for the concrete compression strength and 15-20 % for the bending strength of structural timber. The characteristic value is generally chosen as the 5 % quantile.

Variable loads (imposed and environmental) can be modeled in different ways, see JCSS [12]. The simplest model is to use a stochastic variable modeling the largest load within the reference period (often one year). This variable is typically modeled by an extreme distribution such as the Gumbel distribution. The coefficient of variation is typically in the range 20-40 % and the characteristic value is chosen as the 98 % quantile in the distribution function for the annual maximum load.

Permanent loads are typically modeled by a Normal distribution since it can be considered as obtained from many different contributions. The coefficient of variation is typically 5-10 % and the characteristic value is chosen as the 50 % quantile.

Model uncertainties are in many cases modeled by a Lognormal distributions if the they are introduced as multiplicative stochastic variables and by Normal distributions if the they are modeled by additive stochastic variables. Typical values for the coefficient of variation are 3-15% but should be chosen very carefully. The characteristic value is generally chosen as the 50 % quantile.

Ad 5.
The partial safety factors \( \gamma \) are calibrated such that the reliability indices corresponding to \( L \) different vectors \( \mathbf{p}_j \) are as close as possible to a target probability of failure \( P_F \) or equivalently a target reliability index \( \beta_i = -\Phi^{-1}(P_F) \). This can be formulated by the following optimization problem

\[
\min_{\gamma} \quad W(\gamma) = \sum_{j=1}^{L} w_j \left( \beta_j(\gamma) - \beta_i \right)^2
\]

(25)

where \( w_j \), \( j = 1, \ldots, L \) are factors (\( \sum_{j=1}^{L} w_j = 1 \)) indicating the relative frequency of appearance / importance of the different design situations. Instead of using the reliability indices in (25) to measure the deviation from the target, for example the probabilities of failure can be used:
\[
\min_{\gamma} W'(\gamma) = \sum_{j=1}^{L} w_j (P_{Fj}(\gamma) - P_{Fj}^t)^2
\]  

(26)

where \( P_{Fj}^t \) is the target probability of failure in the reference period considered. Also, a nonlinear objective function giving relatively more weight to reliability indices smaller than the target compared to those larger than the target can be used.

The above formulations can easily be extended to include a lower bound on the reliability or probability of failure for each failure mode.

**Ad 6.**

The optimal partial safety factors are obtained by numerical solution of the optimization problem in step 5. The reliability index \( \beta_j(\gamma) \) for combination \( j \) given the partial safety factors \( \gamma \) is obtained as follows. First, for given partial safety factors \( \gamma \) the optimal design is determined.

If the number of design variables is \( N = 1 \) then the design \( z^* \) can be determined from the design equation, see (17)

\[
G_j(x_c,p_j,z,\gamma) \geq 0
\]  

(27)

If the number of design variables is \( N > 1 \) then a design optimization problem can be formulated:

\[
\begin{align*}
\min \quad & C(z) \\
\text{s.t.} \quad & c_i(x_c,p_j,z,\gamma) = 0, i = 1,...,m_e \\
& c_i(x_c,p_j,z,\gamma) \geq 0, i = m_e + 1,...,m \\
& z^l_i \leq z_i \leq z^u_i, i = 1,...,N
\end{align*}
\]  

(28)

\( C \) is the objective function and \( c_i, i = 1,...,m \) are the constraints. The objective function \( C \) is often chosen as the weight of the structure. The \( m_e \) equality constraints in (28) can be used to model design requirements (e.g. constraints on the geometrical quantities) and to relate the load on the structure to the response (e.g. finite element equations). Often equality constraints can be avoided because the structural analysis is incorporated directly in the formulation of the inequality constraints. The inequality constraints in (28) ensure that response characteristics such as displacements and stresses do not exceed codified critical values as expressed by the design (27). The inequality constraints may also include general design requirements for the design variables. The lower and upper bounds, \( z^l_i \) and \( z^u_i \), to \( z_i \) in (28) are simple bounds. Generally, the optimization problem (28) is non-linear and non-convex.

Next, the reliability index \( \beta_j(\gamma) \) is estimated by FORM/SORM or simulation on the basis of the limit state equations (16) using the optimal design \( z^* \) from (27) or (28).

**Ad 8.**

As discussed above a first guess of the partial safety factors is obtained by solving these opti-
mization problems. Next, the final partial safety factors are determined taking into account current engineering judgment and tradition.

Examples of reliability-based code calibration can be found in Nowak [25], Sørensen et al. [26] and SAKO [27].

6. Discussion

Code calibration using principles of structural reliability and/or taking basis in decision theoretical principles highly facilitates the development of rational and cost efficient design basis in consistency with the prevailing uncertainties. The formal basis for reliability based code calibration in fact exists and has so far for more than a decade. Since then various additional developments have taken place of which maybe the most important are those concerned with the assessment of target failure probabilities. However, the basic technique has essentially remained the same.

In summary, the important items to be considered in consistent, reliability-based code calibration are a broad, international understanding on 1) selection of stochastic models for the uncertain parameters (distribution types and quantification of coefficient of variations), 2) choice of target reliability levels, which have to be related to the stochastic model, recommended reliability levels and/or reliability levels in existing well proven structural codes. The optimal reliability levels could be based on cost optimal, decision theoretical considerations. 3) code formats, incl. load combination rules, 4) selection of structures / failure modes covering the application area of the code. These items need general recommendations especially for national code committees.

Code calibration in practice, however, is not always a straightforward task. The main reason for this being that in order to obtain a consistent code calibration the assessment of design values or more specifically, partial safety factors and load combination factors needs to be performed jointly in essentially one process. This necessity in turn poses strong requirements to the organizational aspects of the code calibration process, which may be manageable on a national scale but which has proven to be rather difficult on an international scale. Due to the above-mentioned organizational problems reliability based code calibration may in effect despite the numerous theoretical and conceptual advantages lead to situations where the resulting design codes become inconsistent.

In order to benefit from reliability based code calibration it is of utmost importance that consistency is achieved and maintained between all steps in the probabilistic representation of uncertainties and limit state functions across the application domain of the code in terms of types of structures, load conditions and materials. In addition to this a strict consistency must be maintained between acceptance criteria and the applied probabilistic modeling. These aspects often pose problems in practice when e.g. for reasons of tradition some partial safety factors are selected by choice in accordance with previous codes and afterwards combined with partial safety factors derived on the basis of reliability analysis. This in turn also has serious implications when attempting to establish a consistent basis for design supported by experiments.

7. References
