Calibration of Reliability Elements for a Column

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Abstract
The basic European document, EN 1990 Basis of structural design, enable to make national decision concerning alternative design procedures and values of various reliability elements. Simple example of a structural member made of different materials shows, that the three alternatives provided in the EN 1990 for combination of actions lead to the inconsistent reliability levels, which may further differ from those provided by the nationally implemented prestandard ENV 1991-1. Presented study of a column made of different materials indicates that the partial factors for actions and material properties should be adjusted to obtain a harmonized reliability level.

Keywords: Partial factors, reliability of a column, reliability index, reliability level.

1 Introduction
Presently the fundamental standard EN 1990 Basis of Structural Design [1] together with other Eurocodes 1 to 9 offer an opportunity for calibration of safety elements in 19 Member States of CEN and other associated countries. It is expected that each country will develop National Annexes to Eurocodes providing sufficient national level of reliability.

EN 1990 [1] allows alternative procedures, values and recommendations with notes where national choice may be made by means of Nationally Determined Parameters. The national choice is intended to be through specific climatic data, different values of reliability elements, values where only symbols are given in Eurocodes and through alternative design procedures. Furthermore, it is possible to decide about application of informative annexes and references to non-contradictory additional information.

Three alternative relationships are given in EN 1990 [1] for combination of actions in the ULS. Moreover, allowance is made for modification of the combination of actions due to the geographical reasons. During the period of co-existence between Eurocodes and existing national standards, each country is expected to make its own comparative structural analyses and calibration of reliability elements for the achievement of national safety level. Not only reliability elements (e.g. values of partial factors for actions and material properties, importance factors), but also the alternative load combinations and design procedures are to be considered during national calibration.

The aim of this contribution is to show some results of the reliability analysis of safety elements for a basic structural member made of different materials. Presented analysis is intended to be used as a part of background materials for the national comparison of reliability of various structural members in the Czech Republic. The
studies conducted in the Klokner Institute will be supplemented by the comparison of designed structures using both suites of existing Czech standards and Eurocodes during the period of co-existence.

2 Design according to Eurocodes

The design condition for a structural member can be in common cases expressed as

$$ R_d \geq E_d $$

where $E_d$ and $R_d$ are the design values for action effects and member resistance [2]. As an example, a column made of five different materials (concrete, steel, composite material, timber and masonry) was designed taking into account design procedures of new Eurocodes EN [1, 3 to 7], currently recommended values of reliability elements and alternative procedures for combination of actions in the ULS.

2.1 Action effects

Alternative combinations of actions are introduced in Section 6 of EN 1990 [1] through the equation (6.10), couple of equations (6.10a, 6.10b) and alternative (6.10amod, 6.10b). The combinations of actions may be based on not more than two variable actions for common cases of buildings with respect to the recommendations of EN 1990 [1].

Thus, the combination of a permanent action $G$ with two variable actions, imposed load $Q$ (considered in this study as a leading variable action) and wind $W$ (accompanying action) are taken into account for the determination of action effects:

A. Considering the formulae (6.10) of EN 1990 [1], the design value of action effect $E_d$ is given as

$$ E_d = \gamma_G G_k + \gamma_Q Q_k + \gamma_W W_k $$

B. Taking into account the couple of equations (6.10a) and (6.10b), the design value of action effect $E_d$ is given as

$$ E_d = \gamma_G G_k + \gamma_Q \psi Q_k + \gamma_W \psi W_k $$

$$ E_d = \xi \gamma_G G_k + \gamma_Q Q_k + \gamma_W \psi W_k $$

The less favourable action effect from (3) and (4) is taken into account.

C. The third alternative procedure differs from the procedure B in the equation (3) where only permanent load is considered (eq. 6.10amod). Thus, the simplified formulae is given as

$$ E_d = \gamma_G G_k $$

The less favourable action effect from (4) and (5) is taken into account.
2.2 Resistance of structural member

The design resistance $R_d$ of the column made from different materials is specified according to the recommendations given in Eurocodes 2 to 6 [3 to 7]. Only centrally loaded column is considered in the following analysis.

2.2.1 Reinforced concrete column

Design resistance of a short reinforced concrete column is given as

$$R_d = A_s f_{sk} / \gamma_s + 0.8 A_c (\alpha f_{ck} / \gamma_c)$$

(6)

where

- $A_s$ is an area of reinforcement
- $A_c$ is an area of concrete
- $f_{sk}$ is a characteristic yield strength of reinforcement
- $f_{ck}$ is a characteristic compressive strength of concrete
- $\gamma_s, \gamma_c$ are partial factors for reinforcement and concrete
- $\alpha$ is a coefficient for long-term effect depending on the compressive strength and the way the load is applied.

Eurocode 1992-1 [3] recommends for concrete $\gamma_c = 1.5$ and for reinforcement $\gamma_s = 1.15$.

2.2.2 Steel column

Design buckling resistance of a steel column in compression is given as

$$R_d = \kappa A_a f_{yk} / \gamma_{ml}$$

(7)

where

- $A_a$ is a cross-sectional area
- $f_{yk}$ is a characteristic yield strength
- $\kappa$ is a factor for buckling depending on the relative slenderness ratio $\bar{\lambda}$
- $\gamma_{ml}$ is a partial factor for steel in buckling for Class I cross-section

Eurocode 1993-1-1 [4] recommends $\gamma_{ml} = 1.15$. Reduction of resistance $R_d$ by factor $\kappa$ is considered taking into account the slenderness effects.

2.2.3 Composite column

Design buckling resistance of a composite column in compression is given as

$$R_d = \kappa (A_a f_{yk} / \gamma_m + 0.85 A_c f_{ck} / \gamma_c + A_s f_{sk} / \gamma_s)$$

(8)

where all the basic variables are mentioned in the text above. Recommended values of partial factors for steel and concrete are following recommendations [3, 4]. Composite column considered in the analyses is made of cylindrical steel profile filled with concrete only ($A_s = 0$).

2.2.4 Timber column

Design buckling resistance of a timber member in compression is given as

$$R_d = A \kappa c k_{mod} f_{ck} / \gamma_n$$

(9)
where $A$ is a cross-sectional area

$k_{\text{mod}}$ is a modification factor taking into account the effect of the duration of load and moisture content

$f_{\text{ck}}$ is a characteristic strength of timber

$\gamma_m$ is a partial factor for timber (recommended $\gamma_m = 1.3$ in [6])

$\kappa_c$ is an instability factor

Reduction of resistance $R_d$ by the factor $\kappa_c$ is considered, following procedure of [6].

### 2.2.5 Masonry column

Design resistance of a masonry column is given as

$$N_{Rd} = \Phi f_k / \gamma_m t b$$

where

$t$ is an effective thickness of the masonry column

$\Phi$ is the capacity reduction factor

$b$ is the width

$f_k$ is the characteristic strength of masonry

$\gamma_m$ is a partial factor for masonry (recommended for units of category 1 in a range from 1.5 to 2.7 depending on the particular class [7])

Characteristic compressive strength for selected solid clay masonry units of Group 1 and cement mortar is given as

$$f_k = K f_b^{0.7} f_m^{0.3}$$

where

$f_b$ is a normalised compressive strength of units

$f_m$ is a compressive strength of mortar

$K$ is a constant depending on the type of mortar and masonry units.

### 3 Reliability analysis

### 3.1 Limit state function

Reliability analysis is based on the limit state function $g(E, R)$ corresponding to the design relationship given in equation (1)

$$g(\theta_E E, \theta_R R) = \theta_R R - \theta_E E$$

where $E$ and $R$ are the vectors of random variables for action effects and resistance of a structural member, $\theta_R$ model uncertainty of the resistance model and $\theta_E$ the coefficient expressing the uncertainty of the action effect.

The following simplified probabilistic models of basic variables indicated in Table 1 were used for the evaluation of the rules for combination of actions using Turkstra’s combination rule.
3.2 Models of basic variables

The probabilistic models of actions are related to their characteristic values $X_k$ used for the determination of design values of actions, see Table 1. The permanent action is described by the normal distribution, variable actions by Gumbel distribution. The compressive strength of material property is described by lognormal distribution. The coefficients of variation are estimated using working materials of JCSS and other background materials, e.g. [9].

The probabilistic models of compressive strength are assessed from the characteristic value $f_k$ using the following expression

$$\mu = f_k/(1-k \cdot w)$$

Table 1. Probabilistic models of basic variables for time invariant reliability analysis.

<table>
<thead>
<tr>
<th>Category of variables</th>
<th>Name of basic variables</th>
<th>Sym. $X$</th>
<th>Dimension</th>
<th>Distribution</th>
<th>Mean $\mu_X$</th>
<th>St. dev. $\sigma_X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actions</td>
<td>Permanent</td>
<td>$G$</td>
<td>kN</td>
<td>N</td>
<td>$G_k$</td>
<td>0,1$\mu_G$</td>
</tr>
<tr>
<td></td>
<td>Imposed - 5 years</td>
<td>$Q$</td>
<td>kN/m$^2$</td>
<td>GU</td>
<td>0,2$Q_k$</td>
<td>1,1$\mu_Q$</td>
</tr>
<tr>
<td></td>
<td>Imposed - 50 years</td>
<td>$Q$</td>
<td>kN/m$^2$</td>
<td>GU</td>
<td>0,6$Q_k$</td>
<td>0,35$\mu_Q$</td>
</tr>
<tr>
<td></td>
<td>Wind - 1 year</td>
<td>$W$</td>
<td>kN/m$^2$</td>
<td>GU</td>
<td>0,5$W_k$</td>
<td>0,4$\mu_W$</td>
</tr>
<tr>
<td></td>
<td>Wind - 50 years</td>
<td>$W$</td>
<td>kN/m$^2$</td>
<td>GU</td>
<td>0,7$W_k$</td>
<td>0,25$\mu_W$</td>
</tr>
<tr>
<td>Model uncertainty</td>
<td>Action effect factor</td>
<td>$\theta_E$</td>
<td>-</td>
<td>N</td>
<td>1,00</td>
<td>0,10</td>
</tr>
<tr>
<td>Resistance</td>
<td>Concrete column</td>
<td>Yield strength</td>
<td>$f_s$</td>
<td>MPa</td>
<td>LN</td>
<td>$f_{sk}+2\sigma_X$</td>
</tr>
<tr>
<td></td>
<td>Compressive strength</td>
<td>$f_c$</td>
<td>MPa</td>
<td>LN</td>
<td>$f_{ck}+2\sigma_X$</td>
<td>5</td>
</tr>
<tr>
<td>Model uncertainty</td>
<td>Resistance factor</td>
<td>$\theta_R$</td>
<td>-</td>
<td>N</td>
<td>1,00</td>
<td>0,15$\mu_\theta$</td>
</tr>
<tr>
<td>Steel column</td>
<td>Cross-sectional area</td>
<td>$A_a$</td>
<td>m$^2$</td>
<td>N</td>
<td>$\mu_A$</td>
<td>0,02$\mu_A$</td>
</tr>
<tr>
<td></td>
<td>Reduction factor</td>
<td>$\kappa$</td>
<td>-</td>
<td>N</td>
<td>$\mu_\kappa$</td>
<td>0,05$\mu_\kappa$</td>
</tr>
<tr>
<td>Model uncertainty</td>
<td>Resistance factor</td>
<td>$\theta_R$</td>
<td>-</td>
<td>N</td>
<td>1,10</td>
<td>0,07$\mu_\theta$</td>
</tr>
<tr>
<td>Composite column</td>
<td>Resistance factor</td>
<td>$\theta_R$</td>
<td>-</td>
<td>N</td>
<td>1,10</td>
<td>0,15$\mu_\theta$</td>
</tr>
<tr>
<td>Timber column</td>
<td>Modification factor</td>
<td>$k_{mod}$</td>
<td>-</td>
<td>-</td>
<td>0,9</td>
<td>0,1$\mu_k$</td>
</tr>
<tr>
<td></td>
<td>Instability factor</td>
<td>$\kappa_c$</td>
<td>-</td>
<td>-</td>
<td>$\mu_\kappa$</td>
<td>0,08$\mu_\kappa$</td>
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<tr>
<td></td>
<td>Cross-sectional area</td>
<td>$A$</td>
<td>m$^2$</td>
<td>N</td>
<td>$\mu_A$</td>
<td>0,05$\mu_A$</td>
</tr>
<tr>
<td></td>
<td>Compressive strength</td>
<td>$f_c$</td>
<td>MPa</td>
<td>LN</td>
<td>20</td>
<td>0,3$\mu_f$</td>
</tr>
<tr>
<td>Model uncertainty</td>
<td>Resistance factor</td>
<td>$\theta_R$</td>
<td>-</td>
<td>N</td>
<td>1,10</td>
<td>0,10$\mu_\theta$</td>
</tr>
<tr>
<td>Masonry column</td>
<td>Thickness</td>
<td>$t$</td>
<td>m</td>
<td>N</td>
<td>0,59</td>
<td>0,01</td>
</tr>
<tr>
<td></td>
<td>Length</td>
<td>$b$</td>
<td>m</td>
<td>Det</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Eccentricity</td>
<td>$e$</td>
<td>m</td>
<td>Det</td>
<td>$\mu_e$</td>
<td>0,01</td>
</tr>
<tr>
<td></td>
<td>Compressive strength of masonry</td>
<td>$f$</td>
<td>MPa</td>
<td>LN</td>
<td>$\mu_f$</td>
<td>0,20$\mu_f$</td>
</tr>
<tr>
<td>Model uncertainty</td>
<td>Resistance factor</td>
<td>$\theta_R$</td>
<td>-</td>
<td>N</td>
<td>1,00</td>
<td>0,15$\mu_\theta$</td>
</tr>
</tbody>
</table>
where the value of the coefficient of variation $w_f$ depends on the used structural material, coefficient $k$ on the knowledge of distribution. The probabilistic models given in Table 1 can be refined on the basis of new information.

The coefficients of model uncertainties $\theta_E$ and $\theta_R$ are described by random variables having the coefficients of variation $\theta_E$ of 0.10 and $\theta_R$ from 0.07 to 0.15.

### 3.3 Results of analysis for a column

The aim of this analysis is to prepare background documents facilitating national decision about alternative design procedures and values of safety elements recommended in EN 1990 [1]. For determination of the influence of permanent and variable actions on the reliability of the structural elements, the characteristic values of actions $G_k, Q_k$ and $W_k$ are expressed by means of the load ratios $\chi$ and $k$ given as

\[
\chi = \frac{Q_k + W_k}{G_k + Q_k + W_k} \quad \text{and} \quad k = \frac{W_k}{Q_k}
\]

which are used as auxiliary variables in the reliability study of the column made of different materials.

Three alternative combinations of actions A to C are considered in the analysis following recommendations of EN 1990 [1]. The alternative D is similarly as alternative A based on the equation (6.10), however, considering reduced values of partial factors recommended in the implemented Czech prestandard ENV 1991-1 [10] (considering factors $\gamma_G = 1.2$, $\gamma_Q = \gamma_W = 1.4$). The results of reliability analysis are shown in the following Figures 1 to 12.

For the column made of different materials, Figures 1, 5, 6, 9 and 10 illustrate the relationship of the reliability index $\beta$ versus load ratio $\chi$; for a load ratio $k = 0$ (effects of imposed load only). Effects of both variable loads (for the load ratio $k = 0.5$) on the reliability of a reinforced concrete column are shown in Figure 2. In this case, the reliability of the element under the influence of both variable actions is increasing slowly.

Figure 3 indicates for a reinforced concrete column the relationship of reliability index $\beta$ versus partial factors for permanent and variable actions (for load ratios $\chi = 0.3$ and $k = 0$). Figure 4 shows the relationship of $\beta$ versus partial factors for material properties (for load ratios $\chi = 0.3$ and $k = 0$). The recommended level 3.8 made in white colour and the surface determined by analysis indicate the boundary facilitating the selection of partial factors. The values of partial factors currently recommended in EN 1990 [1] fulfill the chosen safety level 3.8. However, nationally reduced values of partial factors for actions may lead to the lower safety level of the structural member.

Figure 9 shows that almost the same reliability as in case A may be achieved by alternative D if reduced values of partial factors for actions and increased values of material properties are considered (partial factors $\gamma_G = 1.2$, $\gamma_Q = \gamma_W = 1.4$ for actions and $\gamma_m = 1.3$ in EN 1995-1-1 [6], $\gamma_m = 1.45$ for materials in the implemented prestandard ENV 1995-1-1 in the Czech Republic).
Figure 1. Reliability index $\beta$ versus load ratio $\chi$, for ratio $k = 0$ and combinations A to D for a concrete column.

Figure 2. Reliability index $\beta$ versus load ratio $\chi$, for ratio $k = 0.5$ and combinations A to D for a concrete column.
Figure 3. Reliability index $\beta$ versus partial factors for permanent $\gamma_G$ and variable actions $\gamma_Q$ for a concrete column (load combination B, and load ratios $\chi = 0.3, k = 0$ are considered).

Figure 4. Reliability index $\beta$ versus partial factors for concrete $\gamma_c$ and steel $\gamma_s$ for a concrete column (load combination B and load ratios $\chi = 0.3, k = 0$ are considered).

Figure 5. Reliability index $\beta$ versus load ratio $\chi$, for ratio $k = 0$ and combinations A to D for a steel column.
Figure 6. Reliability index $\beta$ versus load ratio $\chi$, for ratio $k = 0$ and load combinations A to D for a composite column.

Figure 7. Reliability index $\beta$ versus partial factors for permanent $\gamma_G$ and variable actions $\gamma_Q$ for a composite column (load combination B, ratios $\chi = 0,3$ and $k = 0$ are considered).

Figure 8. Reliability index $\beta$ versus partial factor for concrete $\gamma_c$ and steel $\gamma_m$ for a composite column (load combination B, ratios $\chi = 0,3$ and $k = 0$ are considered).
Figure 9. Reliability index $\beta$ versus load ratio $\chi$, for ratio $k = 0$ and load combinations A to D for a timber column.

Figure 10. Reliability index $\beta$ versus load ratio $\chi$, for ratio $k = 0$ and load combinations A to D for a masonry column.
Figure 11. Reliability index $\beta$ versus increasing eccentricity $e$ of a loading effect, combinations A, B and D are considered.

Figure 12. Reliability index $\beta$ versus load ratio $\chi$, for ratio $k = 0$ and combination A for a column made of various materials.
Figure 11 illustrates that the reliability of a masonry column significantly decreases with increasing eccentricity of the load effects. Presently the document prEN 1996-1-1 [7] does not limit the resulting value of eccentricity.

Figure 12 shows the results of reliability analysis for a column made of different materials, considering the load combination A (eq. 6.10). For the common range of load ratio $\chi$ from 0.2 to 0.6, the reliability index $\beta$ for a column made of concrete or composite material seems to be very close, from 4.0 to 4.3, for masonry from 3.9 to 4.0, for steel from 4.1 to 4.5 and for timber from 3.8 to 4.0. The reliability of a column made of different materials is greater than the recommended value $\beta_t = 3.8$ in EN 1990 [1]. It follows from the analysis that the smallest reliability has the column made of timber due to the uncertainties of material properties. The scatter of reliability indexes $\beta$ for a considered structural member made of different materials could be diminished on the basis of partial factor calibration. However, also other basic structural members and other failure modes should be taken into account to obtain satisfactory results for the selected design procedure.

4 Discussion

Results of presented reliability study for a column made of different materials indicate

- alternative A (eq. 6.10) gives for the most common cases of the load ratio $\chi$ (in the range from 0.2 to 0.6) sufficient reliability level, however rather uneconomic;
- alternative B (eq. 6.10a and 6.10b) gives reliability level less than for alternative A, still in most cases above the recommended value of $\beta_t = 3.8$;
- alternative C (eq. 6.10a_mod and 6.10b) gives reliability less than the recommended value of $\beta_t = 3.8$;
- alternative D (eq. 6.10 and nationally selected reliability elements) gives in most cases reliability less than the recommended value of $\beta_t = 3.8$.

The choice of nationally determined parameters is rather complex as it is necessary to take into account also national decision about imposed loads (through the choice of imposed loads and densities specified in ranges in EN 1991-1-1 [11], national workmanship or properties of used materials. Moreover, probability models of some materials facilitating reliability analyses are currently under development.

The presented analysis of a column confirm the results of the previous study [12] that the reliability of the structures, designed according to the documents of CEN may be greater than the reliability obtained using the current national standards and even the prestandards ENV. Applied probabilistic procedures provide methods for readjustement of partial factors for actions and material properties enabling to obtain a harmonised reliability level.

The expected national choice of the fundamental load combination for the permanent and transient design situations and selection of the values of partial factors might considerably influence the national reliability of structures and economic consequences. Further analyses are needed for the preparation of the National annexes to EN 1990 for the Member States of CEN.
5 References


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