Probabilistic Models for Seismic Design and Assessment of RC Structural Walls

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1) Introduction
• Goal of seismic performance based engineering (SPBE) is to attain Civil engineering facilities of **defined** and **predictable** seismic performance.
Three Major Components in SPBE

Demand (Modeling)

Capacity (Damage)

GM
\[ P(\text{Failure}) = P(\text{Demand} > \text{Capacity}) \]

**Seismic Fragility**
Conditional Probability of failure

\[ P(D > C \mid GMI) \]

**Seismic Hazard**
Annual frequency of exceedance

\[ \nu(gmi > GMI) \]

\[ \nu(F) = -\int P(D > C \mid GMI) \frac{d\nu(gmi > GMI)}{d(GMI)} \ d(GMI) \]
2) New Measure of Seismic Ground Motion Severity
Are PGA or Elastic Response Spectra reliable measures of inelastic response?

\[ \Delta_{\text{max}} = 0.12 \text{ m} \]

GM Acc. (g)

\[ C_y = 0.35 \text{ g} \]

PGA=1.30 g

Morgan Hill EQ (Coyote)

\[ S_a = 1.70 \text{ g} \]

PGA=0.52 g

Turkey EQ (Erzincan)

\[ S_a = 0.74 \text{ g} \]

\[ S_a = 0.74 \text{ g} \]

\[ \xi = 0.05 \]

\[ T = 0.6 \text{ sec} \]

\[ \Delta_{\text{max}} = 0.12 \text{ m} \]

GM Acc. (g)
Short period SDOF bilinear systems with different periods and different displacement ductilities

<table>
<thead>
<tr>
<th></th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>$S_a ~ C_y$</td>
<td>0.56</td>
</tr>
</tbody>
</table>
Significant Peak Ground Acceleration (SPGA)

GM Acc. (g)

GM Vel. (m/sec)

SPGA and SVGV

Erzincan (1992)
Short period SDOF bilinear systems with different periods and different displacement ductilities

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_a \sim C_y )</td>
<td>0.56</td>
<td>0.31</td>
</tr>
<tr>
<td>( SPGA \sim C_y )</td>
<td>0.86</td>
<td>0.74</td>
</tr>
</tbody>
</table>
Relationships between $S_a$ & $C_y$ and between SPGA & $C_y$ (T=0.7sec, $\mu=4$ and $\xi=0.05$)
Are PGA or Elastic Response Spectra reliable measures of inelastic response?

- **Morgan Hill EQ (Coyote)**
  - PGA = 1.30 g
  - $S_a = 1.70$ g
  - $T = 0.6$ sec
  - $\xi = 0.05$
  - $\Delta_{\text{max}} = 0.12$ m
  - $C_y = 0.35$ g

- **Turkey EQ (Erzincan)**
  - PGA = 0.52 g
  - $S_a = 0.74$ g
  - $T = 0.6$ sec
  - $\xi = 0.05$
  - $\Delta_{\text{max}} = 0.12$ m
  - $C_y = 0.35$ g

PGA or Elastic Response Spectra: Do they serve as reliable measures of inelastic response?
3) Seismic Capacity
• At LSPL capacity depends on time histories of applied forces and displacements (stresses and strains).

• In practice, however, design and assessment of structures are carried out at section or element levels.

• Therefore, for simplicity, capacity of a structural wall is estimated in terms of its flexural and shear strength and displacement capacities.
• Making use of mechanical behavior, available experimental data and engineering judgments in Bayesian parameter estimating technique, different capacity models are developed.
\[ y = \hat{g}(x, \theta) + \varepsilon \]

Generic Model

Observable Variables

Model Parameters

Variable to be predicted

Model Error
Model Error

- Zero mean (unbiased model)
- Normally distributed (normality assumption)
- Constant standard deviation (homoskedasticity assumption)
Model Error

• In order to approximately satisfy normality and homoskedasticity assumptions, parametric transformation proposed by Box and Cox can be used

\[
y = \frac{z^\lambda - 1}{\lambda} \quad \text{if } \lambda \neq 0
\]

\[
y = \log z \quad \text{if } \lambda = 0
\]
3-1) Shear Strength Capacity
Shear Strength of RC Structural Walls

\[ P \rightarrow V \]

\[ \begin{align*}
V_d & \\
V_s & \\
V_{aT} & \\
V_{CZ} & \\
\end{align*} \]
Shear Failure of RC Walls (Monotonic Loading)

- Shear-Tension Failure
- Diagonal-Compression Failure
- Shear-Compression Failure
Shear Modes of Failure

• Shear-tension failure
Shear Modes of Failure

- Diagonal-compression failure (web crushing)
Shear Modes of Failure

• Shear-compression failure
Shear Failure of RC Walls (Cyclic Loading)

Intersection shear-Flexural Cracks
Shear Modes of Failure

• Sliding-shear failure
Shear Modes of Failure

- Shear-tension failure
- Diagonal-compression failure (web crushing)
- Shear-compression failure
- Sliding-shear failure
- Bond failure
Experimental Results

• Test results of 16 structural walls under cyclic loading are used.

• Nine of walls failed in shear and other seven failed in flexure.

• All data are used to develop probabilistic models for shear strength capacity, flexural displacement, shear displacement models of structural walls.
Shear Strength Capacity (Idealized Model)

\[ \hat{V}_{cap} = \left( \beta_1 a_{asp} + \beta_2 \frac{P}{A_g f'_c} \right) \sqrt{f'_c f_{sc}} b l + V_s \]

\[ V_s = \rho_h f_{yh} b l \leq \beta_3 \sqrt{f'_c f_{sc}} b l \]
Shear Strength Capacity (Accounting for Model Error)

\[
\ln(V_{\text{cap}}) = \ln \left[ \left( \beta_1 a_{\text{asp}} + \beta_2 \frac{P}{A_g f_c'} \right) \sqrt{f_c' f_{\text{sc}}} b l + V_s \right] + \varepsilon_{V_c}
\]
Bayesian Parameter Estimation technique

Model Parameters $\Theta = \Theta(\beta_1, \beta_2, \beta_3, \sigma_{Vc})$

Posterior Distribution $f(\Theta) = a L(\Theta) p(\Theta)$

Likelihood Function

Prior Distribution
Likelihood Function

\[(\varepsilon_{Vc})_k = \ln(v_{cap})_k - \ln(\hat{v}_{cap})_k\]

\[L(\beta_1, \beta_2, \beta_3, \sigma_{Vc}) = \prod_{\text{Shear Failure}} \left[ \frac{1}{\sigma_{Vc}} \phi\left( \frac{(\varepsilon_{Vc})_k}{\sigma_{Vc}} \right) \right]\]

\[\times \prod_{\text{Flexural Failure}} \left[ 1 - \Phi\left( \frac{(\varepsilon_{Vc})_k}{\sigma_{Vc}} \right) \right]\]
Prior Distribution

• Non-informative prior distribution

• Uniform distributions for $\beta_1, \beta_2, \beta_3$, and $\ln(\sigma_{Vc})$
The table presents the correlation coefficient matrix for various parameters related to shear strength capacity. The parameters include $\beta_1$, $\beta_2$, $\beta_3$, and $\sigma_{vc}$.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>0.067</td>
<td>0.013</td>
<td>1</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>2.240</td>
<td>0.244</td>
<td>-0.64</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.500</td>
<td>0.010</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma_{vc}$</td>
<td><strong>0.051</strong></td>
<td><strong>0.002</strong></td>
<td>0.00 0.00 1</td>
</tr>
</tbody>
</table>
Shear Strength Capacity (cont)

\[
\frac{V_{\text{meas}} - V_{\text{cal}}}{V_{\text{meas}}}
\]

Test Number

Walls failed in shear
Walls did not fail in shear
3-2) Shear Displacement
Shear Displacement Model

• Experimental data reveals that yielding of the flexural reinforcement not only result in flexural yielding, but also in “shear yielding” (Oesterle, 1984).
$\Delta^3_H = 2\Delta$

$\Delta^1_H = \Delta^1_V = \Delta$

$= \frac{\Delta}{2}$

$+ \frac{\Delta}{2}$

Pure Flexure

Pure Shear
Shear Deformation
Shear Drift Prediction

\[
\frac{(Drift_{s, meas}^{\text{meas}} - Drift_{s, pred}^{\text{pred}})}{Drift_{s, meas}^{\text{meas}}}
\]

Walls failed in shear

Walls failed in flexure

Test Number

0 2 4 6 8 10 12 14 16
3-3) Flexural Displacement Capacity
Flexural Displacement Capacity

\[ \Delta f = e^{\varepsilon \Delta f} \left[ a \Phi_y H^2 + (\Phi_u - \Phi_y) \bar{L}_p (H - \bar{L}_p / 2) \right] \]
Flexural Displacement Capacity (cont)

\[ \Delta f = e^{\varepsilon_{\Delta f}} \left[ a \Phi_y H^2 + (\Phi_u - \Phi_y) \bar{L}_p \left( H - \frac{\bar{L}_p}{2} \right) \right] \]

\[ \Phi_u = (\gamma_1 - \gamma_2 \mu_\Phi) \Phi'_u \]

\[ (\varepsilon_{c}^{\text{max}})_{\text{mod}} = \theta \varepsilon_{c}^{\text{max}} \]
Flexural Displacement Capacity (cont)

\[ \Delta_f = e^{\varepsilon_{\Delta f}} \left[ a \Phi_y H^2 + (\Phi_u - \Phi_y) \overline{L}_p (H - \overline{L}_p / 2) \right] \]

\[ \Phi_u = \left[ \gamma - (0.030\gamma - 0.014) \mu_\Phi \right] \Phi'_u \]

\[ \left( \varepsilon_{c \text{ max}} \right)_{\text{mod}} = \theta \overline{\varepsilon}_{c \text{ max}} \]
Flexural Displacement Capacity

Predicted Top Displ. Capacity (m)

Measured Top Displacement Capacity (m)
4) Seismic Fragility of Short Period RC Structural Walls
Seismic Fragility of Short Period RC Structural Walls

\[ F(\text{SPGA}) = P\left( g_f \leq 0 \cup g_V \leq 0 \mid \text{SPGA} \right) \]

\[ g_f(x, \varepsilon_{My}, \varepsilon_{\Delta f}, \varepsilon_{\Delta s}; \Theta; \text{SPGA}) = (\text{SPGA})_{\text{cap}} - (\text{SPGA})_d \]

Intrinsic Variability

Epistemic uncertainty

\[ g_V(x, \varepsilon_{Vc}, \varepsilon_{Vd}, \varepsilon_{cy}, \varepsilon_{\Delta f}, \varepsilon_{\Delta s}; \Theta; \text{SPGA}) = V_{\text{cap}} - V_d \]
Plan of a Five Story RC Structural Wall System

5m x 5m x 5m
Cross Section of a Five Story RC Structural Wall

Designed based on ACI 318-99
Seismic Fragility of example Short Period RC Structural Wall

![Graph showing seismic fragility](image)

- Probability of Failure vs. SPGA (g)
- Curves indicate different failure modes:
  - Mean
  - Mean Shear Failure
  - Mean Flex. Failure
  - Mean ± S.D. Bound

Workshop on Reliability Based Code Calibration
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CONCLUSIONS

• Incorporating mechanics of shear and flexural behavior of RC walls, using Bayesian parameter estimating technique, and utilizing available experimental data, probabilistic capacity models for flexural deformation, shear strength, and shear deformation of RC structural walls are developed.

• Significant errors observed in some available models in current seismic codes suggest need for accounting for model errors in probabilistic design of structures.