

**JCSS PROBABILISTIC MODEL CODE
PART 3: RESISTANCE MODELS**

3.1 CONCRETE PROPERTIES

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List of symbols:

- f_{co} = basic concrete compression strength
- M_j = the logarithmic mean at strength job j
- Σ_j = the logarithmic strength standard deviation at job j
- $Y_{1,j}$ = a log-normal variable representing additional variations due to the special placing, curing and hardening conditions of in situ concrete at job j
- U_{ij} = a standard normal variable
- λ = lognormal variable with mean 0.96 and coefficient of variation 0.005; generally it suffices to take λ deterministically
- $\alpha(t,\tau)$ = is a deterministic function which takes into account the concrete age at the loading time t and the duration of loading τ
- $\phi(t,\tau)$ = is the creep coefficient.
- β_d = total load and depends from the type of the structure
- E_c = modulus of elasticity
- f_c = in situ strength
- ϵ_c = strain at yielding
- ϵ_u = ultimate strain

3.1.1 Basic Properties

The reference property of concrete is the compressive strength f_{co} of standard test specimens (cylinder of 300 mm height and 150 mm diameter) tested according to standard conditions and at a standard age of 28 days (see ISO/DIS 2736 and ISO 3893). Other concrete properties are related to the reference strength of concrete according to:

$$\text{In situ compressive strength: } f_c = \alpha(t, \tau) f_{co}^\lambda \quad [\text{MPa}] \quad (1)$$

$$\text{Tensile strength: } f_{ct} = 0.3 f_c^{2/3} \quad [\text{MPa}] \quad (2)$$

$$\text{Modulus of elasticity: } E_c = 10.5 f_c^{1/3} \left(\frac{1}{1 + \beta_d \varphi(t, \tau)} \right) \quad [\text{GPa}] \quad (3)$$

$$\text{Ultimate compression strain: } \varepsilon_u = 6.10^{-3} f_c^{-1/6} (1 + \beta_d \varphi(t, \tau)) \quad [\text{m/m}] \quad (4)$$

λ is a factor taking into account the systematic variation of in situ compressive strength and strength of standard tests (see 3.1.3)

$\alpha(t, \tau)$ is a deterministic function which takes into account the concrete age at the loading time t [days] and the duration of loading τ [days]. The function is given by:

$$\alpha(t, \tau) = \alpha_1(\tau) \alpha_2(t) \quad (5a)$$

$$\alpha_1(\tau) = \alpha_3(\infty) + [1 - \alpha_3(\infty)] \exp[-a_\tau \tau] \text{ with } \alpha_3(\infty) \approx 0.8 \text{ and } a_\tau = 0.04.$$

$$\alpha_2(t) = a + b \ln(t) \quad (5b)$$

In most applications $\alpha_1(\tau) = 0.8$ can be used. The coefficients a and b in $\alpha_2(t)$ depend on the type of cement and the climatical environment; under normal conditions $a = 0.6$ and $b = 0.12$.

$\varphi(t, \tau)$ is the creep coefficient according to some modern code assumed to be deterministic.

β_d is the ratio of the permanent load to the total load and depends on the type of the structure; generally β_d is between 0.6 and 0.8.

3.1.2 Stress-strain-relationship

For concrete under compression the following simplified stress-strain relationship holds:

$$\sigma = E_c \varepsilon \quad \text{for } \varepsilon < \varepsilon_c \quad (6)$$

$$\sigma = f_c \quad \text{for } \varepsilon_c < \varepsilon < \varepsilon_u \quad (7)$$

$$\varepsilon_c = f_c / E_c \quad (8)$$

For calculations where the form of the stress-strain relationships is important the following relationship should be used:

$$\sigma = f_c \left[1 - \left[1 - \frac{\varepsilon}{\varepsilon_s} \right]^k \right] \quad (9)$$

$$\varepsilon_s = 0.0011 f_c^{1/6} \quad (10)$$

$$k = \frac{E_c \varepsilon_s}{f_c} \quad (11)$$

The relationship holds for $0 < \varepsilon < \varepsilon_s$.

3.1.3 The probabilistic model

The strength of concrete at a particular point i in a given structure j as a function of standard strength f_{c0} is given as:

$$f_{c,ij} = \alpha(t, \tau) (f_{c0,ij})^\lambda Y_{1,j} \quad (12)$$

$$f_{c0,ij} = \exp((U_{ij} \Sigma_j + M_j)) \quad (13)$$

in which

- $f_{c0,ij}$ = log-normal variable, independent of $Y_{1,j}$, with distribution parameters M_j and Σ_j
- M_j = the logarithmic mean at job j
- Σ_j = the logarithmic standard deviation at job j
- $Y_{1,j}$ = a log-normal variable representing additional variations due to the special placing, curing and hardening conditions of in situ concrete at job j
- U_{ij} = a standard normal variable representing the variability within one structure
- λ = lognormal variable with mean 0.96 and coefficient of variation 0.005; generally it suffices to take λ deterministically

The variable $Y_{i,j}$ can also be taken as a spatially varying random field whose mean value function takes account of systematic influences in space.

Correspondingly, for the other three basic properties:

$$f_{ct,ij} = 0.3 f_{c,ij}^{2/3} Y_{2,j} \quad (14)$$

$$E_{c,ij} = 10.5 f_{c,ij}^{1/3} Y_{3,j} (1 + \beta_d \varphi(t, \tau))^{-1} \quad (15)$$

$$\varepsilon_{u,ij} = 6 \cdot 10^{-3} f_{c,ij}^{-1/6} Y_{4,j} (1 + \beta_d \varphi(t, \tau)) \quad (16)$$

where the variables $Y_{2,j}$ to $Y_{4,j}$ mainly reflect variations due to factors not well accounted for by concrete compressive strength (e.g., gravel type and size, chemical composition of cement and other ingredients, climatical conditions).

The variables U_{ij} and U_{kj} within one member are correlated by:

$$\rho(U_{ij}, U_{kj}) = \rho + (1 - \rho) \exp\left\{-\frac{(r_{ij} - r_{kj})^2}{d_c^2}\right\} \quad (17)$$

where $d_c = 5$ m and $\rho = 0.5$. For different jobs U_{ij} and U_{kj} are uncorrelated.

3.1.4 Distributions of Y_{kj}

Unless direct measurements are available, the parameters of the variables Y_{kj} can be taken from Table 3.1.1. The variables are distributed according to the log-normal distribution. The variability of the variables Y_{kj} can further be split into a part depending only on the job under consideration and a part representing spatial variability.

If direct measurements are available, the parameters in Table 3.1.1 are taken as parameters of an equivalent prior sample with size $n' = 10$ (see Part 1 for the details of updating).

Variable	Distribution type	Mean	Coefficient of variation	Related to
$Y_{1,j}$	LN	1.0	0.06	compression
$Y_{2,j}$	LN	1.0	0.30	tension
$Y_{3,j}$	LN	1.0	0.15	E-modulus
$Y_{4,j}$	LN	1.0	0.15	ultimate strain

Table 3.1.1: Data for parameters Y_i

3.1.5 Distribution for f_{co}

The distribution of $x_{ij} = \ln(f_{co,ij})$ is normal provided that its parameters M and Σ obtained from an ideal infinite sample. In general it must be assumed that concrete production varies from production unit, site, construction period, etc. and that sample sizes are limited. Therefore, the parameters M and Σ must also be treated as random variables. Then, x_{ij} has a student distribution according to:

$$F_x(x) = F_{t_{v''}} \left[\frac{\ln(x/m'')}{s''} \left(1 + \frac{1}{n''}\right)^{-0.5} \right]$$

where $F_{t_{v''}}$ is the Student distribution for v'' degrees of freedom. $f_{co,ij}$ can be represented as

$$f_{co,ij} = \exp\left(m'' + t_{v''} s'' \left(1 + \frac{1}{n''}\right)^{0.5}\right)$$

The values of m'' , n'' , s'' and v'' depend on the amount of specific information. Table 3.1.2 gives the values if no specific information is available (prior information).

Table 3.1.2: Prior parameters for concrete strength distribution (f_{co} in MPa) [1, 2]

Concrete type	Concrete grade	Parameters			
		m''	n''	s''	v''
Ready mixed	C15	3.40	3.0	0.14	10
	C25	3.65	3.0	0.12	10
	C35	3.85	3.0	0.09	10
	C45	3.98	3.0	0.07	10
	C55	-	-	-	-
Pre-cast elements	C15	-	-	-	-
	C25	3.80	3.0	0.09	10
	C35	3.95	3.0	0.08	10
	C45	4.08	4.0	0.07	10
	C55	4.15	4.0	0.05	10

The prior parameters may depend on the geographical area and the technology with which concrete is produced.

If $n''v'' > 10$, a good approximation of the concrete strength distribution is the log-normal distribution

with mean m'' and standard deviation $s'' \sqrt{\frac{n''}{n''-1} \frac{v''}{v''-2}}$.

References

[1] Kersken-Bradley, M., Rackwitz, R., Stochastic Modeling of Material Properties and Quality Control, JCSS Working Document, IABSE-publication, March 1991

[2] Rackwitz, R., Predictive Distribution of Strength under Control, Materials & Structures, 16, 94, 1983, pp. 259 - 267