JCSS PROBABILISTIC MODEL CODE

EXAMPLE APPLICATIONS

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EXAMPLE 1: REINFORCED CONCRETE SLAB

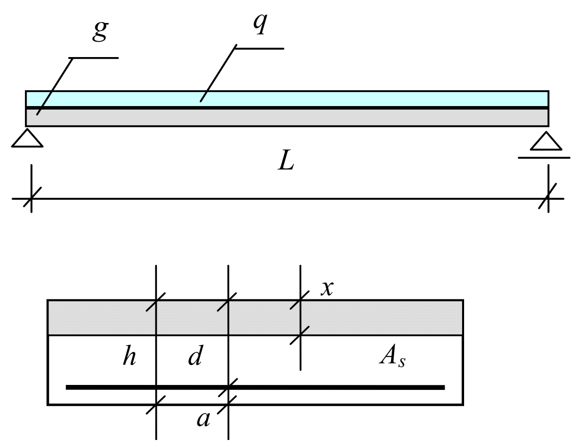

Figure 1.1 Simply supported reinforced concrete slab and its cross-section.

Table 1.1 Probabilistic models for the reinforced concrete slab example (acc. to JCSS Probabilistic Model Code 2001).

Basic variable	Sym-bol	Distr. type	Dimen-sion	Mean	Standard deviation	V	λ	ρ
Compression concrete strength	f_c	lognormal	MPa	30	5	0.17		
Yield strength	f_y	lognormal	MPa	560	30	0.05		
Span of the slab	L	determin.	m	5	-			
Reinforcement area	A_s	determin.	m^2	nom.	-			
Slab depth	h	normal	m	0.2	0.005	0.025		
Distance of bars to the slab bottom	a	gamma	m	$c + \phi/2$	0.005	0.17		
Density of concrete	γ_{con}	normal	MN/m^3	0.025	0.00075	0.03		
Imposed long-term load	q_{lt}	gamma	kN/m^2	0.5	0.75	1.5	0.2/year	perm.
Imposed short-term load	q_{st}	exponenc.	kN/m^2	0.2	0.46	2.3	1/year	1/365
Uncertainty of resistance	θ_R	lognormal	-	1.1	0.077	0.07		
Uncertainty of load effect	θ_E	lognormal	-	1	0.2	0.2		

The simply supported reinforced concrete slab has the span of 5 m and cross-sectional depth of 0.20 m. The slab carries permanent load g and imposed load q (office areas) which cause the bending moment. The model of permanent load is determined as the weight of a concrete floor of a uniform equivalent thickness of 0.25 m (including weight of the slab and floor layers). The following material characteristics for concrete and reinforcing steel are considered: concrete class C 20/25 and reinforcing steel S 500.

The reliability of the designed slab is verified using probabilistic methods. The limit state function Z for slab may be expressed as

$$Z = \theta_{\boldsymbol{R}}\, A_{\mathrm{s}} f_{\mathrm{y}} \left(h - a - 0.5\, A_{\mathrm{s}} f_{\mathrm{y}} / f_{\mathrm{c}} \right) - \theta_{\boldsymbol{E}} \left(g + q_{\mathrm{lg}} + q_{\mathrm{st}} \right) L^2 / 8 \tag{1.1}$$

where a is the axial distance of reinforcement to the slab bottom ($a = c + \phi/2$, c is the concrete cover, ϕ is the diameter). The basic variables applied in the reliability analysis are listed in Table 1.1. Statistical properties of the random variables are further described by the moment characteristics, the mean and standard deviation. Models of variables follows recommendations of JCSS [1]. Some of the basic variables are assumed to be deterministic values (A_{s}, L), while the others are considered as random variables having the normal, lognormal, exponential and Gamma distribution.

Coefficients of model uncertainties $\theta_{\boldsymbol{R}}$ and θ_E are random variables to cover imprecision and incompleteness of the relevant theoretical models for resistance and load effects, imposed load q is assessed by imposed long-term load q_{lt} and imposed short-term load q_{st} for office areas [1].

The mean and standard deviation of the imposed long-term load correspond to the distribution of 5 years maximum. This is expressed in Table 1.1 by means of the renewal rate λ = 1/5. Interarrival-duration intensity ρ is considered as permanent. Following the recommendations of JCSS [1] for office areas the mean of long-term load m = 0.5 kN/m^2, standard deviations $\sigma(v)$ = 0.3 kN/m^2 and $\sigma(u)$ = 0.6 kN/m^2, the reference area A_0 = 20 m^2. The influence area A in this example is assumed to be A = 30 m^2 and factor κ for the shape of influence line i(x,y) κ= 2. Following JCSS [1], Clause 2.2.2 the standard deviation of long-term load is given as

$$\sigma(q_{\mathrm{lg}})(u) = \sqrt{\sigma(v)^2 + \sigma(u)^2 \frac{A_0}{A} \kappa} = \sqrt{0.3^2 + 0.6^2 \frac{20}{30}\, 2} = 0.75 \text{ kN/m}^2 \tag{1.2}$$

For short-term imposed loads the renewal rate λ = 1 (1 occurrence per year) and the interarrival-duration intensity ρ = 1/365 corresponds to the arrival rate and mean duration of one day. For the short-term imposed load JCSS [1] gives m = 0.2 kN/m^2 and $\sigma(u)$ = 0.4 kN/m^2. The standard deviation of the short-term load is assessed as

$$\sigma(q_{st)}(u) = \sqrt{\sigma(u)^2 \frac{A_0}{A}\kappa} \;=\; \sqrt{0.4^2 \frac{20}{30} 2} \;= 0.46 \text{ kN/m}^2 \tag{1.3}$$

The model of the reinforcement cover follows Section 3.10 of JCSS [1], μ = 0.03 nom, σ= 0.005 m (bottom steel), coefficient of variation v = 0.17, gamma distribution.

The models of uncertainty are considered according to Section 3.9 of JCSS [1]. Model uncertainty for the bending moment capacity θ_R has the mean μ = 1.1 and standard deviation σ = 0.077, model uncertainty for the load effect θ_E has the mean μ = 1 and standard deviation σ = 0.2, lognormal distribution.

The software product Comrel [2] is used for time-dependent reliability analysis of the reinforced concrete slab. The reference period of fifty years is taken into account. The increasing reliability index β_1 (lower bound (P_f)) from 1.9 to 4.5 and β_2 (upper bound (P_f)) from 0.4 to 3.9 for the slab depth of 0.20 m depending on designed reinforcement ratio $A_s/[b(h - a)]$, considering this ratio in the interval from 0.2 % to 0.5 %, is shown in Figure 1.2. The reliability index β of the slab is considered to be within the lower and upper bound of reliability indices β_1 and β_2. The target value β_t = 3.8 for fifty year reference period and the ultimate limit states is recommended in common cases in JCSS Probabilistic Model Code [1], in ISO 2394 General principles on reliability for structures and EN 1990 Basis of structural design.

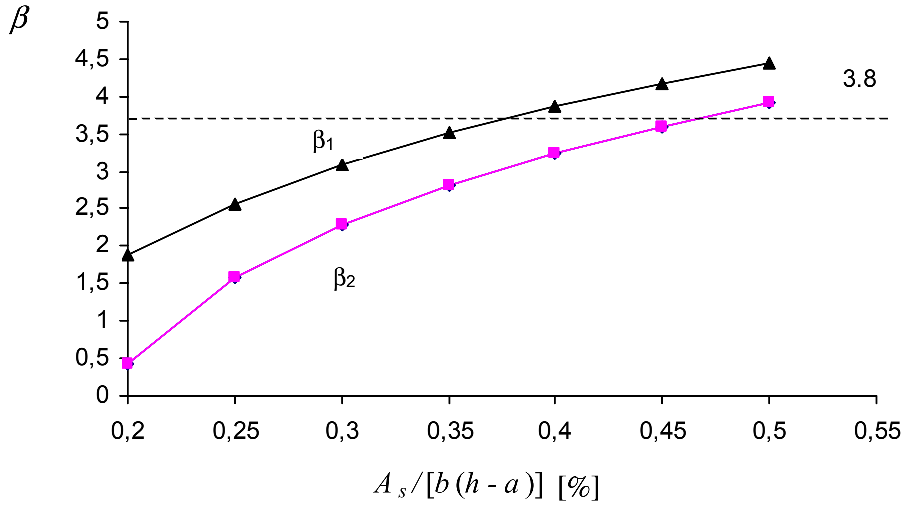

Fig. 1.2 The reliability of reinforced concrete slab versus reinforcement ratio.

The selected sensitivity factors α shown in Table 1.2 express the influence of basic variables to the resulting reliability of reinforced concrete slab. The active imposed load is considered.

Table 1.2 Sensitivity factors α of selected basic variables for active imposed load.

Basic variable	Sensitivity factor α	Basic variable	FORM factor α
cross-sectional height h	0.05	concrete strength in compr. f_c	0.01
distance of bars a	-0.10	yield strength f_y	0.16
imposed load q_{st}	-0.18	uncertainty of resistance θ_R	0.21
imposed load q_{lt}	-0.71	uncertainty of load θ_E	-0.61

References

[1] JCSS Probabilistic Model Code. 2001.
[2] Comrel, RCP Consulting software, version 7.10, Munich, 1999.

EXAMPLE 2: STEEL BEAM

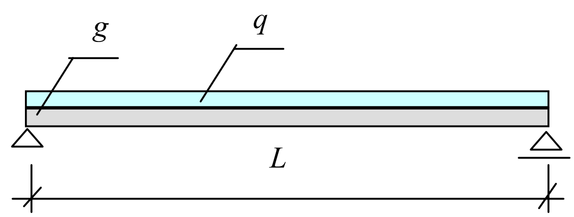

Figure 2.1 Steel beam as a load-bearing floor element in shopping areas.

Table 2.1 Probabilistic models for the steel beam example (acc. to JCSS Probabilistic Model Code 2001).

Basic variable	Symbol	Distr. type	Dimension	Mean	Standard deviation	V	λ	ρ
Yield strength	f_y	lognormal	MPa	280	19.6	0.07		
Span of the beam	L	determin.	m	5	-			
Section modulus	W	determin.	m^3	param.	-			
Concrete density	γ_{con}	normal	MN/m^3	0.024	0.00096	0.04		
Slab depth	h	normal	m	0.25	0.01	0.04		
Distance of beams	d	determin.	m	3	-			
Imposed long-term load categ. D	q_{lt}	gamma	kN/m^2	0.9	2.15		0.2/year	perm.
Imposed short-term load categ. D	q_{st}	exponenc.	kN/m^2	0.4	1.42		1/ year	14/365
Uncertainty of resistance	θ_R	lognormal	-	1	0.05	0.05		
Uncertainty of load effect	θ_E	lognormal	-	1	0.2	0.2		

The simply supported beam of a rolled steel section I is a load-bearing floor element in the shopping areas of the category D [1], its span is 5 m. The beam carries permanent load g due to its self-weight, the weight of concrete slab and floor layers. The distance of beams d = 3 m. The model of the permanent load is considered here as the weight of a reinforced concrete floor of a uniform equivalent thickness of 0.25 m including self-weight of the steel beam. The beam is made of steel grade S 235 (nominal yield strength f_{yk} = 235 MPa).

The reliability of the designed steel beam is verified using probabilistic methods. The limit state function Z for the beam is expressed as

$$Z = \theta_R\, W f_y - \theta_E\, d(g + q_{lg} + q_{st})\, L^2 / 8 \tag{2.1}$$

The basic variables applied in the reliability analysis are listed in Table 2.1. Models of variables follows recommendations of JCSS [2]. Some of the basic variables are assumed to be deterministic values (d, L, W), while the others are considered as random variables having the normal, lognormal, exponential and Gamma distribution.

Coefficients of model uncertainties $\theta_{\boldsymbol{R}}$ and θ_E are random variables to cover imprecision and incompleteness of the relevant theoretical models for resistance and load effects. The imposed load q for shopping areas is assessed by imposed long-term load q_{lt} and imposed short-term load q_{st} following JCSS, Clause 2.2.4 [2] for live load models.

The mean and standard deviation of the imposed long-term load for shopping areas correspond to the distribution in range from 1 to 5 years maximum (here it is assumed 5 years). This is expressed in Table 2.1 by means of renewal rate $\lambda = 0.2$. It is considered permanent inter-arrival duration intensity ρ. Following the recommendations for shopping areas indicated in JCSS [2], the mean of long-term load m = 0.9 kN/m^2, the standard deviations $\sigma(v)$ = 0.6 kN/m^2 and $\sigma(u)$ = 1.6 kN/m^2, the reference area A_0 = 100 m^2. The influence area A is assumed to be A = 120 m^2 in this example and factor κ for the shape of influence line i(x,y) $\kappa = 2$. Following JCSS, Clause 2.2.2 [2] the standard deviation of long-term load is given as

$$\sigma(q_{\mathrm{lg}})(u) = \sqrt{\sigma(v)^2 + \sigma(u)^2\,\kappa \frac{A_0}{A}} = \sqrt{0.6^2 + 1.6^2\,2\frac{100}{120}} = 2.15\ \mathrm{kN/m^2} \tag{2.2}$$

For short-term imposed loads are considered 14 occurrences per year (the range may be from 1 to 14 occurrences per year depending on the shopping process). Thus, the inter-arrival duration intensity ρ = 14/365. For the short-term load m = 0.4 kN/m^2 and $\sigma(u)$ = 1.1 kN/m^2. The standard deviation of the short-term load is assessed as

$$\sigma(q_{\mathrm{st}})(u) = \sqrt{\sigma(u)^2\,\kappa \frac{A_0}{A}} = \sqrt{1.1^2\,2\,\frac{100}{120}} = 1.42\ \mathrm{kN/m^2} \tag{2.3}$$

The models of uncertainty are considered according to Table 3.9.1 of JCSS [2]. The model uncertainty for the bending moment capacity has for steel the mean μ = 1 and standard deviation σ = 0.05, the model uncertainty for the load effect μ = 1 and standard deviation σ = 0.1, lognormal distribution.

The software product Comrel [3] is used for time-dependent reliability analysis of the steel beam. The reference period of fifty years is taken into account. The reliability index β_1 (lower bound (P_f)) increases from 3.1 to 4.9, and β_2 (upper bound (P_f)) increases from 2.3 to 4.3 with the increasing section modulus W of the steel beam (see Figure 2.2). The reliability index β of the beam is assumed to be within the lower and upper bounds of reliability indices β_1 and β_2. Horizontal dash line in Figure 2.2 indicates the recommended reliability index β_t = 3.8 for the ultimate limit states following recommendations of JCSS [2].

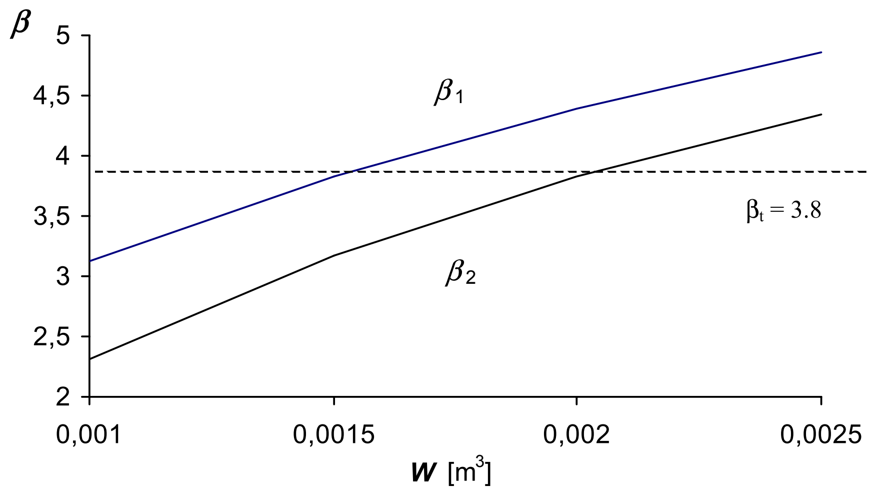

Fig. 2.2 The reliability index β of a steel beam versus section modulus W.

The selected sensitivity factors α shown in Table 2.2 express the influence of basic variables to the resulting reliability of steel beam. The active imposed load is considered.

Table 2.2 Sensitivity factors α of selected basic variables.

Basic variable	Sensit. factor α	Basic variable	FORM. factor α
imposed load q_{st}	-0.06	uncertainty of load θ_E	-0.33
imposed load q_{lt}	-0.92	yield strength f_y	0.16
resistance uncert. θ_R	0.11	concrete density	-0.01

References

[1] prEN 1991 Actions on Structures, Part 1.1 Densities, Self-weight and Imposed Loads on Buildings. European Committee for Standardisation, CEN/TC 250 Final Draft, July 2001.

[2] JCSS Probabilistic Model Code, 2001.

[3] Comrel, RCP Consulting software, version 7.10, Munich, 1999.

EXAMPLE 3: TWO STOREY STEEL FRAME

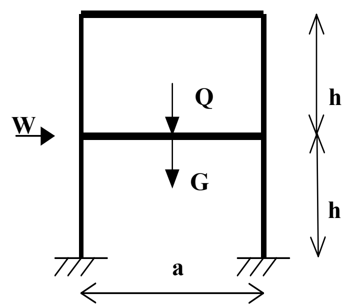

Figure 3.1. Two storey steel frame

Consider the simple two storey steel frame of Figure 3.1. The floors are supposed to be of concrete. Let the limit state function for a particular member failure be given by:

$$Z = R - 0.16\, m_E\, h\, (G + Q + W) \tag{3.1}$$

where R is the resisting bending moment, G the self weight, Q the live load and W the wind load. The factor 0.16 is the result of a structural analysis. The details of that analysis are not relevant for this example. The m_E is the model factor and h the storey height. The resistance R and the forces G, Q and W are respectively given by:

$$R = m_R\, Z_p\, f_y \tag{3.2a}$$

$$G = a\, b\, t\, \rho_c\, g \tag{3.2b}$$

$$Q = a\, b\, (q_{long} + q_{short}) \tag{3.2c}$$

$$W = 2\, h\, b\, c_a\, c_g\, c_r\, (0.5\, m_q\, \rho_a\, U^2) \tag{3.2d}$$

The designation of the variables as well as their deterministic values or probabilistic models as derived from the JCSS Probabilistic Model Code are given in Table 3.1.

Table 3.1 Probabilistic models for the steel frame example (according to the JCSS Probabilistic Model Code 2001)

X	Designation	Distribution	Mean	V	λ
a	in plane column distance	Deterministic	6 m	-	
b	frame to frame distance	Deterministic	5 m	-	
h	storey height	Deterministic	3 m	-	
t	thickness concrete floor slab	Normal	0.20 m	0.03	
Z_p	plastic section modulus	Normal	$0.0007 m^3$	0.02	
f_y	steel yield stress	Lognormal	300 MPa	0.07	
g	acceleration of gravity	Deterministic	$10 m/s^2$	-	
ρ_c	mass density concrete	Normal	$2.4 ton/m^3$	0.04	
q_{long}	long term live load (sustained)	Gamma	$0.50 kN/m^2$	1.15	0.2/year
q_{short}	short term live load (1 day)	Exponential	$0.20 kN/m^2$	1.60	1.0/year
ρ_a	mass density air	Deterministic	$0.125 kg/m^3$	-	
c_a	aerodynamic shape factor	Normal	1.10	0.12	
c_g	gust factor	Normal	3.05	0.12	
c_r	roughness factor	Normal	0.58	0.15	
u	ref wind speed (8 hours)	Weibull	5 m/s	0.60	3.0/day
U	ref wind speed (one year)	Gumbel	30 m/s	0.10	1.0/year
m_q	model factor wind pressure	Normal	0.80	0.20	
m_R	model factor resistance	Normal	1.00	0.05	
m_E	model factor load effect	Normal	1.00	0.10	

The information in Table 3.1 can be derived from the JCSS Probabilistic Model code. For some of the variable some clarification will be given:

The gust factor c_g in the wind model is defined as:

$$c_g(z) = 1 + 2\, g_p\, I_u(z) \qquad (3.3)$$

where z is the building height, g_p is the peak factor and $I_u(z) = 1/\ln(z/z_o)$ the turbulence intensity. The building height in this example is equal to two times the storey height, so $z = 2h$. The peak factor g_p for a storm period of 8 hours is about 4.2. Finally the roughness parameter z_0 is assumed to be 0.10 m. The coefficient of variation depends primarily on the variability of g_p.

The roughness factor c_r is given by:

$$c_r(z) = 0.8\left[\frac{\ln z - \ln z_0}{\ln z_{ref} - \ln z_{oref}}\right]^2 \left(\frac{z}{z_{oref}}\right)^{0.07} \qquad (3.4)$$

The reference values z_{ref} and $z_{0\text{ref}}$ have the standard values 10 m and 0.03 m respectively. The factor 0.8 is an average correction factor. For the coefficient of variation of the roughness model the JCSS code recommends 0.15.

For the steel resistance the Model Code gives:

$$\mu(f_y) = f_{ynom}\exp(1.64V) - 20 \text{ MPa} \quad \text{and } V = 0.07 \qquad (3.5)$$

Starting from a nominal yield stress of 290 MPa we derive at $\mu(f_y)$ = 300 MPa. Normally for steel no job specific tests are performed, so the "gross supply estimates" for $\mu(f_y)$ and V are used.

For the long term live load the model code gives for offices m = 0.50 kN/m^2, $\sigma(v)$ = 0.30 kN/m^2, $\sigma(u)$ = 0.60 kN/m^{2,} and A_o = 20 m^2. The influence area in this example is $A = 2ab$ = 60 m^2 and let κ = 2. In that case we have:

$$\sigma(q_{long}) = \sqrt{\{\sigma(v)^2 + \sigma(u)^2\,\kappa Ao/A\}} = 0.57 \text{ kN/m}^2 \qquad (3.6)$$

According to the code the average renewal time is 5 years. This is expressed in the last column of Table 3 by means of the renewal rate λ = 1 / 5 years.

For the short term load the model code gives m = 0.2 kN/m^2 and $\sigma(u)$ = 0.4 kN/m^2. In that case we find:

$$\sigma(q_{long}) = \sqrt{\{\sigma(u)^2\,\kappa Ao/A\}} = 0.33 \text{ kN/m}^2 \qquad (3.7)$$

According to the code the average renewal time for the short term is one year and each time the duration of the short term load is 1 day. For the wind speed u the Model Code recommends a 2 parameter Weibull distribution to describe the daily fluctuations. A certain wind condition is supposed to last for about 8 hours. For yearly extremes the Gumbel distribution is recommended. For both distributions the parameters depend on the local wind climate.

Given these data, the failure probability for a design life time of 50 years can be determined as follows. First the probability that the structure fails for a period of 5 years is calculated, assuming the short term live load to be absent all the time. The loading scheme, presented as FBC models, is given in Figure 3.2. The floor load Q (sustained part) is defined for a period of 5 years (λ = 0.2/yr), so that is okay. The wind speed distribution in Table 3.1 is of the Gumbel type and defined as a one year extreme. The maximum for a 5 year period can be found by raising the mean of the one year period according to μ [5 yr] = μ [1 yr] + 0.78 σ ln(5) = 34 m/s. The standard deviation σ of the wind does not change. There is no need to adjust the distribution of the permanent variables.

Given these data, the failure probability for an assumed design lifetime of 50 years can be determined. We will follow a simplified procedure, comprising out of two load cases:

Load case 1: Self weight, Long term live load and Wind

The short-term live load will be neglected in this part of the analysis. First the probability is calculated that the structure fails in a period of 5 years, being the average renewal period of the long-term live load. This means that we can directly use the data from Table 3.1 for the live load, the self-weight and the resistance. The wind speed distribution in Table 3.1 is defined for the one year extreme, so an adjustment to 5 years has to be made. According to the theory of extreme values the mean value for the 5 year extreme should be taken as μ [5 years] = 30 m/s + 0.78 σ ln(5) = 34 m/s. A standard time independent FORM analysis for this case leads to a reliability index β = 4.1 and a failure probability P_F = 2.3 10^{-5}. For the assumed design life of 50 years, using a simple upper bound approximation for convenience, we find P_F = 10 * 2.3 10^{-5} = 2.3 10^{-4}. The FORM influence coefficients α can be found in Table 3.2

Load case 2: Self weight, Long and Short term live load and Wind.

Next we look at a single day where the short-term load is active. So now the short-term floor load is present in the limit state equation with the distribution as given in Table 3.1. For the long-term load we also take the distribution from the table. The wind speed model assumes an FBC model with Δt = 8 hrs, so we have to consider the maximum of three Weibull distributed variables with parameters as given in the table. Using standard FORM again we arrive at β = 4.4 and P_F = 5.0 10^{-6}. Recall that this result holds for a single day with the short-term floor load present. According to the model code there is one such a day every year, so during the design period of 50 years we have 50 days of short-term live load activity. An upper bound approximation, neglecting correlation due to resistance parameters and the permanent and sustained loads, gives P_F = 2.5 10^{-4}.

Load combination and checking with the target

Adding conservatively the two results for both cases one arrives at P_F = (2.3 + 2.5) 10^{-4} = 4.8 10^{-4} or P_F = 3.3 for the life time. As an average over the lifetime we have a nominal failure rate of P_F = 10^{-5} per year corresponding to β = 4.3. Looking at the model code this seems quite acceptable and we consider the design to be satisfactory.

Of course, this analysis could haven been performed in a more accurate way. In principle, the Model Code recommends the Outcrossing Approach to deal with time fluctuating phenomena.

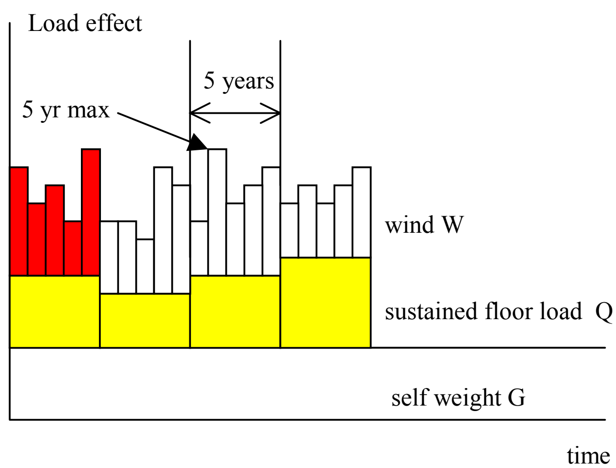

Figure 3.2: Self weight, sustained floor load and wind load (yearly maximum) as function of time based on FBC models.

Table 3.2 Probabilistic influence coefficients (Alfa values) from the FORM analysis for the case with the short term load absent

X	**Designation**	α
Z_p	plastic section modulus	0.12
f_y	steel yield stress	0.41
ρ_c	mass density concrete	-0.20
t	thickness concrete floor slab	-0.15
q_{long}	long term live load (sustained)	-0.65
c_a	aerodynamic shape factor	-0.02
c_g	gust factor	-0.02
c_r	roughness factor	-0.03
U	ref wind speed (one year)	-0.01
m_q	model factor wind pressure	-0.03
m_R	model factor resistance	0.37
m_E	model factor load effect	-0.42

EXAMPLE 4: REINFORCED CONCRETE COLUMN IN MULTI STOREY FRAME

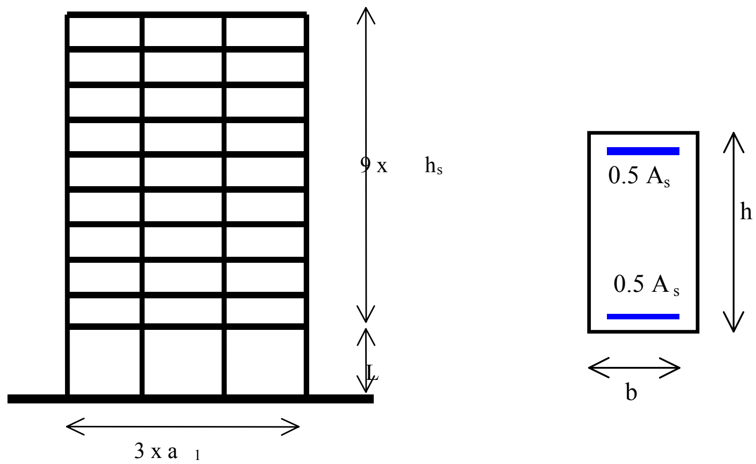

Figure 4.1. Multistorey Concrete frame

The multi-storey concrete structure considered in the this study is schematically shown in Figure 4.1. Each plenary frame in the transversal direction of the structure may be considered as unbraced sway frame. These transversal sway frames consist of four columns at a constant distance a_1; in the longitudinal direction of the structure they are located within a constant distance a_2. The edge bottom column having the height L and rectangular cross section with h/b = 2 is considered. The column is considered as fully clamped in at top and bottom end. The axial column force N is considered as a simple sum of axial forces due to all the considered actions:

$$N = N_{\mathrm{W}} + N_{\mathrm{imp}} + N_{\mathrm{wind}} \qquad (4.1)$$

where N_{W} is the axial force due to self weight, N_{imp} is the axial force due to the long and/or short imposed load and N_{wind} is the axial force due to wind action (positive values correspond

Table 4.1 Probabilistic models for the concrete column example (acc to JCSS Model Code 2001)

X	Designation	Distribution	Mean	V	λ
a_1	column distance in plane	deterministic	5 m	-	
a_2	perpend. dist. of column	deterministic	5 m	-	
t	plate thickness (conventional)	deterministic	0.30 m	0.03	
h_s	storey height	deterministic	3 m	-	
L	height of bottom column	deterministic	6 m	-	
n	number of floors	deterministic	10	-	
b	width of cross section	normal	350 mm	0.007	
h	height of cross section	normal	700 mm	0.014	
A_s	reinforcement area	deterministic	0.01 bh	-	
$d_{1(2)}$	distance of bars from edge	normal	75 mm	0.07	
ζ	initial overall sway(1)	normal	0	σ=15 mrad	
g	acceleration of gravity	deterministic	10 m/s^2	-	
ρ_c	mass density concrete	normal	2.4 ton/m^3	0.04	
q_{long}	long term live load (sustained)	gamma	0.50 kN/m^2	0.75 (1)	0.2/year
q_{short}	short term live load (1 day)	exponential	0.20 kN/m^2	1.60	1.0/year
ρ_a	mass density air	deterministic	1.25 kg/m^3	-	
c_a	aerodynamic shape factor	normal	1.10	0.12	
c_g	gust factor	normal	4.06	0.12	
c_r	roughness factor	normal	0.45	0.15	
u	ref wind speed (8 hours)	weibull	4 m/s	0.60	3.0/day
U	ref wind speed (one year)	gumbel	24 m/s	0.10	1.0/year
f_c	concrete strength (C35)	logstudent	30 MPa	0.18 (2)	
α	long term reduction factor	normal	0,85	0,10	
f_y	yield strength	normal	560 MPa	0.06	
E	modulus of elasticity for steel	deterministic	200 GPa	-	
m_q	model factor wind pressure	normal	0.80	0.20	
m_R	uncertainty of column	normal	1,10	0,11	
m_E	model factor load effect	normal	1.00	0.10	

(1) Including the effect of correlation over the various floors (ρ=0.5)

(2) More precisely: the logarithm of f_c in MPa has a Student distribution with m=3.85 s=0.12, n=3 and ν=6

to compressive forces):

$$N_W = (n + 1)\, a_1\, a_2\, t\, \rho_c \;/\; 2 \tag{4.2}$$

$$N_{imp} = n\, a_1\, a_2\, q_{imp} \;/\; 2 \tag{4.3}$$

$$N_{wind} = (1/2)(L + nh_s)^2\, a_2\, c_a\, c_g\, c_r\, m_q\, q_{ref} / (3\, a_1) \tag{4.4}$$

For the designation of all the variables as well as for their values and probabilistic models, reference is made to Table 4.1. In this analysis the weight of beams and columns are incorporated in the plate thickness for convenience. In (4.4) q_{ref} stands for $0.5\ \rho_a\ U^2$. The bending end moment M is given by:

$$M = M_0 + N(\, e_a + e_2) \tag{4.5}$$

where M_0 is the first order moment. Assuem that th total horizontal wind force

$$W = c_a\, c_g\, c_r\, m_q\, q_{ref}\ (L+nh_s)\, a_2$$

is taken equally by the 4 columns, leading to a bending moment of 0.5 WL at top and bottom. In that case the wind part of the first order moment is

$$M_0 = L[c_a\, c_g\, c_r\, m_q\, q_{ref}\ (L+nh_s)\, a_2]/8 \tag{4.6}$$

The first order moments by self weight are zero if we sum them up over the four columns. As a consequence there is no need to take them into account if plastic redistribution is assumed. A similar argument holds for the contribution of the imposed load. Therefore, (4.6) represents the total first order bending moment M_0.

The additional eccentricity e_a and the second order eccentricity e_2 (according to the Eurocode 2, Design of Concrete Structures) in (4.5) are given by:

$$e_a = \zeta L/2 \tag{4.7}$$

$$e_2 = 0{,}2\ L^2\ K_2 f_y \,/\, (0{,}9\, E_s\, (h - d_1)) \tag{4.8}$$

where the initial sway ζ, and K_2 is given by (according to Eurocode 2):

$$K_2 = (N_u - N) / (N_u - N_{bal}) \leq 1 \quad (4.9)$$

In K_2 the symbol N stands for the normal force according to (4.1); N_u and N_{bal} are respectively given by $N_u = \alpha bhf_c + A_s f_y$ and $N_{bal} = \alpha bh\, f_c/2$. The limit state function Z for the right hand loweredge column may be expressed as the difference of the resistance moment and the load induced end moment about the centroid:

$$Z = m_R\, M_R - m_E\, M \quad (4.10)$$

The two coefficients m_R and m_E are the model uncertainties. The bending moment M is according to (4.5). Using a calibrated approximation for the resistance model according to Eurocode 2, we can elaborate M_R to:

$$M_R = [A_s f_y(h-2d_1)/2 + hN(1-N/(2\alpha bhf_c)] \quad (4.11)$$

$$M_R = K_2[A_s f_y(h-2d_1)/2 + \alpha bh^2 f_c/8\,] \quad (4.12)$$

for $N < \alpha bhf_c / 2$ and for $N > \alpha bhf_c / 2$ respectively. All basic variables applied in the model are listed in Table 4.1.

Given these data, the failure probability for a design life time of 50 years can be determined as follows. First the probability that the structure fails for a period of 5 years is calculated, assuming the short term live load to be absent all the time. For a more detailed explanation, see Example 1. The wind speed distribution in Table 4.1 is defined for the one year extreme, so an adjustment has to be made. According to the theory of extreme values the mean value should be taken as μ = 30 m/s + 0.78 σ ln(5) = 34 m/s. A FORM analysis for this case leads to a reliability index β = 3.8 and a failure probability P_F = 6.9 10^{-5}. The alfa-values are presented in Table 4.2. For a period of 50 years, using a simple upperbound approximation, we find P_F = 6.9 10^{-4}. In this case the short term live load is of little significance, which means that the final result is P = 1.4 10^{-5} per year, corresponding to β = 3.6. This is an acceptable result according to the target values in Part 1, Basis of Design.

Table 4.2 Probabilistic influence coefficients (Alfa values) from the FORM analysis for the case with the short term load absent

X	**Designation**	α
t	plate thickness (conventional)	0.01
b	width of cross section	0.08
h	height of cross section	0.10
$d_{1(2)}$	distance of bars from edge	0.07
ζ	initial overall sway[(1)]	0.55
ρ_c	mass density concrete	0.02
q_{long}	long term live load (sustained)	0.02
c_a	aerodynamic shape factor	-0.19
c_g	gust factor	-0.19
c_r	roughness factor	-0.22
U	ref wind speed (one year)	-0.40
f_c	concrete strength (C35)	0.03
α	long term reduction factor	0.02
f_y	yield strength	-0.05
m_q	model factor wind pressure	0.28
m_R	uncertainty of column	0.56
m_E	model factor load effect	-0.35