JCSS PROBABILISTIC MODEL CODE
PART 2: LOAD MODELS

2.0 GENERAL PRINCIPLES

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2.0 GENERAL PRINCIPLES

2.0.1 Introduction

The environment in which structural systems function gives rise to internal forces, deformations, material deterioration and other short-term or long-term effects in these systems. The causes of these effects are termed actions. The environment from which the actions originate can be of a natural character, for example, snow, wind and earthquake. lt can also be associated with human activities such as living in a domestic house, working in a factory, etc.

The following concepts of actions are used in this document.

1) An action is an assembly of concentrated or distributed forces acting on the structure. This kind of action is also denoted by "load".
2) An action is the cause of imposed displacements or thermal effects in the structure. This kind of action is often denoted by "indirect action".
3) An action is an environmental influence which may cause changes with time in the material properties or in the dimensions of a structure.

Action descriptions are in most cases based on suitably simple mathematical models, describing the temporal, spatial and directional properties of the action across the structure. The choice of the level of richness of details is guided by a balance between the quality of the available or obtainable information and a reasonably accurate modelling of the action effect. The choice of the level of realism and accuracy in predicting the relevant action effects is, in time, guided by the sensitivity of the implied design decisions to variations of this level and the economical weight of these decisions. Thus the same action phenomenon may give rise to several very different action models dependent on the effect and structure under investigation.

2.0.2 Classifications

Loads can be classified according to a number of characteristics. With respect to the type of the loads, the following subdivision can be made:

- self weight of structures
- occupancy loads in buildings, e.g. loads from persons and equipment
- actions caused by industrial activities, e.g. silo loads
- actions caused by transport: traffic, liquids in pipelines, cranes, impact, etc.
- climatic actions, e.g. snow. wind, outdoor temperature etc.
- hydraulic actions, e.g. water and ground water pressures
- actions from soil or rock, including earth quake

This classification does not cover all possible actions hut most of the common types of actions can be included in one or more classes. Some of the classes belong as a whole either to uncontrollable actions or to controllable actions. Other actions may belong to both e.g. water pressure.

With respect to the variations in time the following classification can be made:

- **permanent actions,** whose variations in time around their mean is small and slow (e.g. self weight, earth pressure) or which monotonically a limiting value (C.g. prestressing, imposed deformation from construction processes, effects from temperature, shrinkage, creep or settlements)
- **variable actions,** whose variations in time are frequent and large (e.g. all actions caused by the use of the structure and by most of the external actions such as wind and snow)
- **exceptional** actions, whose magnitude can be considerable but whose probability of occurrence for a given structure is small related to the anticipated time of use. Frequently the duration is short (e.g. impact loads, explosions, earth and snow avalanches).

As far as the spatial fluctuations are concerned it is useful to distinguish between fixed and free actions. **Fixed actions** have a given spatial intensity distribution over the structure. They are completely defined if the intensity is specified in a particular point of the structure (e.g. earth or water pressure). For **free actions** the spatial intensity distribution is variable (e.g. regular occupancy loading, involved although they are variable actions.

2.0.3 Modelling of actions

There are two main aspects of the description of an action: one is the physical aspect, the other is the statistical aspect. In most cases these aspects can be clearly separated. Then the physical description gives the types of physical data which characterise the action model, for example, vertical forces distributed over a given area. The statistical description gives the statistical properties of the variables, for example, a probability distribution function. In some cases the physical and statistical aspects are so integrated that they cannot be considered separately.

A complete action model consists in general, of several constituents which describe the magnitude, the position, the direction, the duration etc. of the action. Sometimes there is an interaction between the components. There may in certain cases also be an interaction between the action and the response of the structure.

One can **in** many cases distinguish between two kinds of variables (constituents) Fo and W describing an action F (see also part 1, Basis of Design).

$$F = \varphi (F_O, W) \qquad (2.0.3.1)$$

F_o is a basic action variable which is directly associated with the event causing the action and which should be defined so that it is, as far as possible, independent of the structure. For example, for snow load F_o is the snow load on ground, on a flat horizontal surface

W is a kind of conversion factor or model parameter appearing in the transformation from the basic action to the action F which affects the particular structure. W may depend on the form and size of the structure etc. For the snow load example W is the factor which transforms the snow load on ground to the snow load on roof and which depends on the roof slope, the type of roof surface etc.

$\varphi(-)$ is a suitable function, often a simple product.

The time variability is normally included in F_o, whereas W can often be considered as time independent. A systematic part of the space variability of an action is in most cases included in W, whereas a possible random part may be included in F_o or in W. Eq. (2.0.3.1) should be regarded as a schematic equation. For one action there may be several variables F_o and several variables W.

Any action model contains a set of parameters and variables that must be evaluated before the model can be used. In probabilistic modelling all action variables are in principle assumed to be random variables or processes while other parameters may be time or spatial co-ordinates, directions etc. Sometimes parameters may themselves be random variables, for example when the model allows for statistical uncertainty due to small sample sizes.

An action model often includes two or more variables of different character as is described by eq. (2.0.3.l). For each variable a suitable model should be chosen so that the complete action model consists of a number of models for the individual variables.

These models may be described in terms of:

- stochastic processes or random fields
- sequences of random variables
- individual random variables
- deterministic values or functions

The definition of the models for these quantities require probability distributions (see annex 2) and a description of the correlation patterns.

2.0.4 Models for fluctuations in time

2.0.4.1 Types of models

To describe time depended loads, one needs the probability distribution for the “arbitrary point in time values" and a description of the variations in time. Some typical process models are (see figure 2.0.4.l):

a) Continuous and differentiable process
b) Random sequence
c) Point pulse process with random intervals
d) Rectangular wave process with random intervals
e) Rectangular wave process with equidistant intervals Δ

If the load intensities in subsequent time intervals of model (e) are independent, the model is referred to as a **FBC model** (Ferry Borges Castanheta model).

In many applications a combination of models is used, e.g. for wind the long term average is often modelled as an FBC model while the short term gust process is a continuous Gaussian process. Such models are referred to as **hierarchical models** (see Part 1, Basis of Design, Section 5.4). Each term in such a model describes a specific and independent part of the time variability. For a number of further definitions and notions, reference is made to Annex 1.

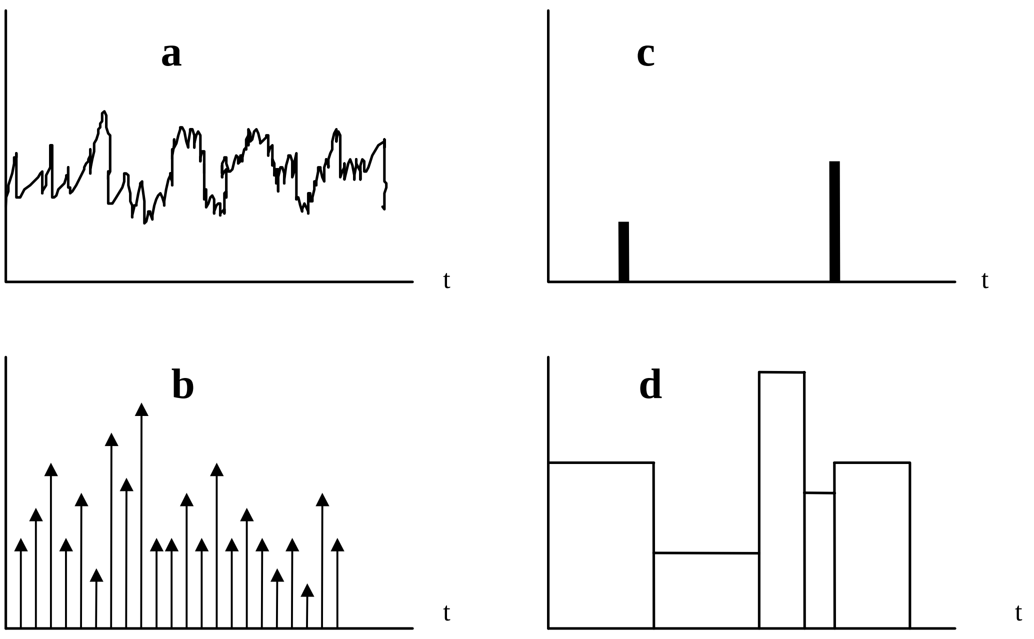

Figure 2.0.4.1: Various types of load models

2.0.4.2 Distribution of extremes for single processes

At the design the main interest is normally directed to the maximum value of the load in some reference period of time t_o. A quite general and useful upperbound formula to calculate the distribution of the maximum is given by:

$$F_{\max F}(a) \cong \exp[-t_o \, \nu^+(a)] \qquad (2.0.4.1)$$

The upcrossing frequency $\nu^+(a)$ is given by:

$$\nu^+(a) = P\{F_t < a \text{ and } F_{t+dt} > a\} / dt \qquad (2.0.4.2)$$

For the FBC model $\nu^+(a)$ is simply given by:

$$\nu^+(a) = (1-F_F(a))\, F_F(a) / \Delta t \cong (1-F_F(a))/\Delta t \qquad (2.0.4.3)$$

And for a continuous Gaussian process:

$$\nu^+(a) = \frac{1}{2\pi}\sqrt{-\rho''(0)} \quad \exp(-\beta^2 / 2) \qquad (2.0.4.4)$$

where $\beta = (a-\mu(F)) / F(F)$ and ρ = the correlation function.

2.0.4.3 Distribution of extremes for hierarchical processes

Consider the case that the load model contains slowly and rapidly varying parts, as well as random variables that are constant in time (see figure 2.0.4.2).

$$F = R + Q + S \qquad (2.0.4.5)$$

R = random variables, constant in time
Q = slow rectangular process with mean renewal rate λ
S = fast varying process

In that case the following expression (see Annex 3, A.3.5) can be used:

$$F_{max\,F}(a) = E_R \left[\exp[\lambda t_o[1-E_Q \exp(-\Delta t\, \nu_s^+ (a|RQ))]] \right] \qquad (2.0.4.6)$$

$\nu_s^+(a|RQ)$ = upcrossing rate of level "a" for process S, conditional upon R and Q
$\Delta t = 1/\lambda$ = time interval for the rectangular process Q

E_R and E_Q denote the expectation operator over all variables R and Q respectively.

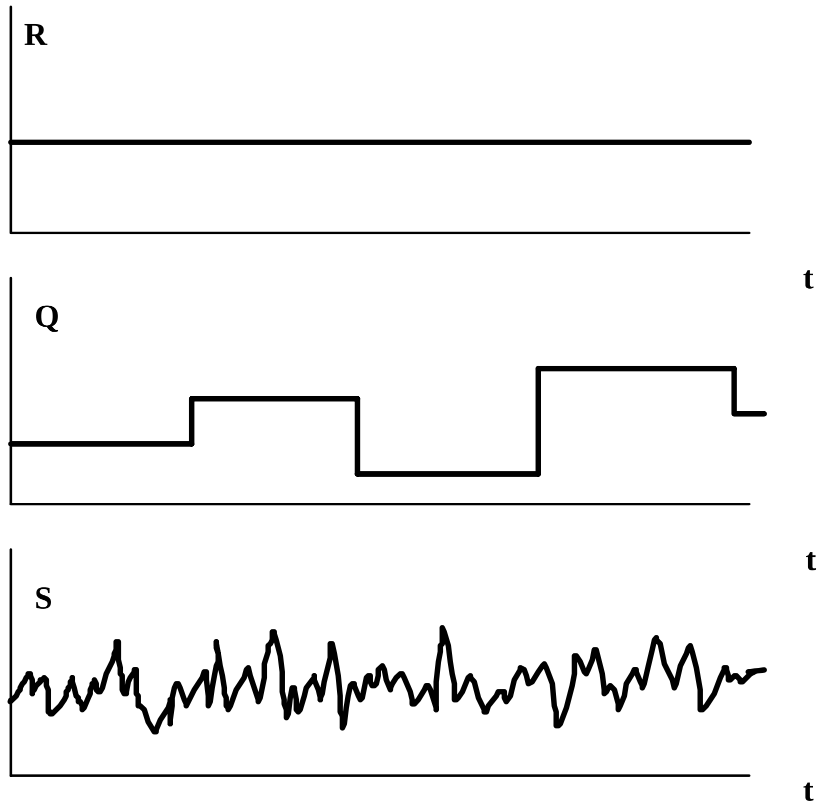

Figure 2.0.4.2: Hierarchical model for time dependent loads

2.0.5 Models for Spatial variability

2.0.5.1 Hierarchical models

As an example for the spatial modelling of actions using a hierarchical model consider the live load in an office building:

$$F = m + \Delta F_1 + \Delta F_2 + \Delta F_3(x,y) \qquad (2.0.5.1)$$

where:

m is a general mean value for the whole population

ΔF_1 is a stochastic variable which describes the variation **between** the load on different floors. The distribution function for ΔF_1 has the mean value zero and the standard deviation σ_1

ΔF_2 is a stochastic variable which describes the variation **between** the load in rooms on the same floor but with different floor areas. The distribution function for ΔF_2 has the mean value zero and the standard deviation σ_2

ΔF_3 is a random field which describes the spatial variability of the load **within** a room.

The total variability of the samples taken from the total population is described by $\Delta F_1 + \Delta F_2 + \Delta F_3$. The variability **within** the subpopulation of floors is described by $\Delta F_2 + \Delta F_3$.

2.0.5.2 Equivalent uniformly distributed load (EUDL)

Consider a simple hierarchical distribution load model given by:

$$q(x,y) = q_o + q_{loc}(x,y) \qquad (2.5.0.2)$$

q_o = the variability between the various structures or structural elements
q_{loc} = the small scale or point to point fluctuation.

In many cases the random field q is replaced by a so called Equivalent Uniformly Distributed Load (EUDL). This load is defined as:

$$q_{EUDL}(t) = \frac{\int q(x,y,t)i(xy)dA}{\int i(xy)dA} \qquad (2.5.0.3)$$

when i(x,y) is the influence function for some specific load effect (e.g. the midspan bending moment).

For given statistical properties of the load field q(x,y) the mean and standard deviation of q_{EUDL} can be evaluated. For a homogeneous field, that is a random field where the statistical properties of q(x,y) do not depend on the location, we give here the resulting formulas:

$$\mu(qEUDL) = \mu(q_o) \qquad (2.5.0.4)$$

$$\sigma^2(q_{EUDL}) = \sigma^2(q_o)+\sigma^2(q_{loc})\int\int\int\int i(x,y)i(\xi,\eta)\rho(d)dxdyd\xi d\eta/[\int\int i(x,y)dxdy]^2 \qquad (2.5.0.5)$$

Here $\rho(d)$ is the correlation function describing the correlation between the small scale load q_{loc}, on the two points (x,y) and (ξ,η). This function may be of the form:

$$\rho(\Delta r) = \exp\{-\Delta r^2 / d_c^2\} \qquad (2.5.0.6)$$

with $\Delta r^2 = (x-\xi)^2 + (y-\eta)^2$, Δr being the distance between the two points, and d_c some scale distance. The correlation function tends to zero for distances Δr much larger than d_c.

If the field can be schematised as an FBC-field, the formula for $\sigma^2(q_{EUDL})$ can be simplified to:

$$\sigma^2(q_{EUDL}) = \sigma^2(q_i) + \sigma^2(q_{loc})\, \kappa A_o/A \qquad (2.5.0.7)$$

Here A_o is the reference area of the FBC-field and A stands for the total area under consideration, the so called tributary area. The formula is valid only for $A>A_o$.

The parameter κ is a factor depending on the shape of the influence line i(x,y). Values are presented in Figure 2.5.0.1. The figure κ = 1 corresponds to a constant value of i(x,y).

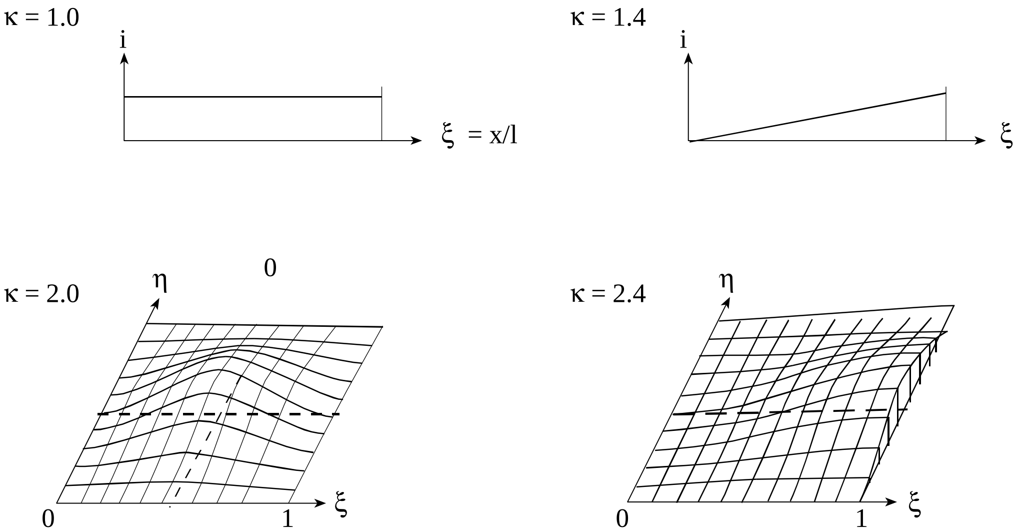

Figure 2.5.0.1: Random fields and corresponding κ-values.

2.0.6 Interactions and correlations between actions

For describing dependencies between various actions it is useful to distinghuish between:

-actions of the same nature
-actions of different nature

Actions of the same nature are for instance floor loads on different floors in one building or the wind loads on the front and back wall. The combination of wind and snow is a typical example of the combinatin of actin of a differente type. Note that sometimes it may be less clear: it may be difficult to decide whether floor loads of a completely different type in one building (say office loads and storage loads) are loads of the same nature of a different nature.

If the actions are of *the same nature*, one might better consider them as components of one action, The various components are normally described by similar probabilistic models. The basic question is then to model the statistical dependency between the processes. In general this is a purely mathematical problem. Details of the mathematical description of the dependencies depend on the nature of the physical relationship and the nature of the processes themselves. One possibility is to construct a hierarchical model as has been explained in section 2.5.1. For two stationary continuous Gaussian processes x(t) and y(t) the correlation alternatively may be described by the cross correlation function $R_{xy}(\tau)$ or alternatively by the cross spectrum $S_{xy}(\omega)$ (see Annex 1). For pulse type processes we may have to distinguish between the correlation in amplitude, arrival time and duration. Floor loads in multi-storey buildings are a good example where all three correlations are of importance.

If the actions are *of a different nature*, they sometimes may show quite complex physical interactions. Typical examples are:

Snow and wind
If snow and wind act together, the result may be that the wind reduces the accumulated snow load on the roof. For some building configuration, however, the combined action by wind and snow may result in much higher loads on some specific locations. This dependency between wind and snow is present even if wind and snow are statistically completely independent processes (which is not the case). In such cases we need a more complex model 2.0.3.1 where the final load is calculated as a function of both wind speed and snow intensity. In addition one may need a statistical correlation between wind and snow as compontens of the same multicomponenet atmospheric system.

Earthquake and fire
Earthquakes are often followed by fire: due to the damage of the pipes and heating systems gas may come free and a fire may be started. The earth quake is said to act as a triggger mechanism for fire. In order to treat this interaction properly, one should consider

1. the probability of a fire starting given an earth quake has occured and
2. the probability of collapse, given earth quake and fire

The second analysis should take into account that all extinction devices may be not working and that the structure already may be damamed by the earth quake.

Additional to that, of course, one still needs to consider the standard cases of collapse under earth quake alone and collapse under fire alone.

Wind and traffic on bridges
Traffic on bridges enlarges the wind load, but heavy wind will reduce the traffic. One may need a model expressing the wind force given traffic and wind speed and a model expressing the conditional probabability density of the traffic intensity as a function of wind speed.

So in all above examples one need to build a more advanced physical model on the one hand and conditional probability models of one load given the (extreme) condition of the other. In most cases it may be convenient to define one of the processes as the “leading one” and describe arrival times and amplitudes of the second process conditional upon the occurrence and amplitude of the first one.

In this model code none or little guidance is presented to this matter. However, the user of this model code is always entitled to be aware of these possible correlations and interactions. It is stressed that these interactions may be of great importance to the reliability of the structure.

2.0.7 Combinations of actions

From a mathematical modelling point of view the load on a structure is a joint set F(t) of time varying random fields. This set of loads gives a rectorial load effect E(t) in a given cross section or point of the structure at time t as a function of F(t) (i.e. a random process). In the scalar case we have:

$$E(t) = c_1\, F_1(t) + c_2\, F_2(t) + .. \qquad (2.0.7.1)$$

The reliability problem related to the considered point is to evaluate the probability P_f that $F_{max}(t)$ E V for all future time V is the nonfailure domain defined by the strength properties at the considered point and limit state.

The load combination problem is to formulate a reasonably simple but for the considered engineering purpose sufficiently realistic mathematical model that defines **F**(t). The needed level of detailed modelling of **F**(t) depends on the filtering effect of the function that maps **F**(t) into the load effect E(t). This filtering effect is judged under due consideration of the sensitivity of the probability p_f to the detailing of the models. The sensitivity question is tied to the last part of the load combination problem which is actually to compute the value of P_f. Thus, to be operational, the modelling of **F**(t) should be simple enough to enable at least a computer simulation of the scalar process E(t) to an extent that allows an estimation of P_f.

First the relevant set of different action types is identified. This identification defines the number of elements in the set **F**(t) and the subdivision of **F**(t) into stochastically independent subsets. The modelling is next concentrated on each of these subsets with dependent components.

The mathematical difficulty to solve probabilities for outcrossing rates of processes of the type (2.0.7.1) is the possible very different nature of the various contributors F_i . Each of these processes may be of a completely different nature, including all kinds of continuos and intermittent processes. Numerical solutions will often prove to be necessary, but also analytical solutions may prove to be very helpful. Reference is made to Annex 3 and to the literature.

ANNEX 1 - DEFINITIONS

Covariance function

The covariance function r (t_1, t_2) is defined by:

$$r(t_1, t_2) = E[(F(t=1) - m_1)(F(t_2) - m_2)]$$

$$m_1 = E[F(t_l)] \qquad m_2 = E[F(t_2)]$$

Stationary processes

The process is defined for $-\infty < t < \infty$. If, for all values t_1 and for all values τ**,** chosen such that $0 \leq t_i \leq$ to and $0 \leq t_i + \tau \leq t_o$, the stochastic variable x $(t_i + \tau)$ has the same distribution function as the stochastic variable x (t_i) the stochastic process x (t) is stationary.

If the mean value function m (t) is constant and the covariance function r (t_1, t_2) depends solely on the difference $\tau = (t_2 - t_l)$ the process is sold to be wide-sense stationary.

Thus the covariance function for a stationary or a wide sense stationary process may be written

$$r(\tau) = E[(F(t + \tau) - m)(F(t) - m)]$$

The concept of stationary applied to action processes should in most cases be interpreted as wide-sense stationary.

Ergodic processes

A process is ergodic if averaging over several realisations and averaging with respect to time (or another index parameter) give the same result.

For ergodic processes a relation between the point-in-time value distribution function F and the excursion time t is determined for a chosen reference period t_o, by

$$1 - F_F(F) = t/t_o$$

The correlation function

The correlation function for a stationary process is:

$$\rho(\tau) = \frac{r(\tau)}{r(0)}$$

For ergodic processes $\rho(\tau = \infty) = 0$

Spectrum

A stationary stochastic process may be characterised with ald of a spectrum:

$$S(n) = \int_{-\infty}^{\infty} e^{-i2\pi n\tau} r(\tau) d\tau$$

S(n) may be regarded as a measure of how the process is built up of components with different frequencies. The total variance of the process is:

$$\mathrm{Var}\,Q = 2\int_0^{\infty} S(n)dn$$

Gaussian processes

A stochastic process S(t) is a Gaussian process if the multidimensional probability distribution functions for all the stochastic variables $S(t_i)$ are Gaussian. The stochastic properties of a Gaussian process is completely determined by the mean value and the covariance function or by the spectrum.

Scalar Nataf Processes

A special but important class of non-Gaussian, scalar and differentiable processes are built by a memoryless transformation from a normal process, i.e.

$$S(t) = h(U(t))$$

where U(t) is a standard normal process and h(u) is an arbitrary function. For S(t) any admissible (unimodal) distribution function can be chosen thus defining a certain class of functions h(u). In addition the autocorrelation function $\rho_s(t_1,t_2)$ has to be specific. However, there are some restrictions on the type of autocorrelation function.

Scalar Hermite Processes

The Hermite process is a special case of the Nataf process. All marginal distribution must be of Hermite type. For this process the solution of the integral equation occurring for the autocorrelation function of the equivalent (or better generating) standard normal process is analytic. The standard Hermite process has the representation, i.e. a special case of the function h(u)

$$S(t) = \kappa(U(t) + \tilde{h}_{3,i}(U(t)^2 - 1) + \tilde{h}(U(t)^3 - 3U(t)))$$

For the coefficients depending on the first four moments of the marginal distribution of the non-normal process. In addition, the Hermite process requires specification of the autocorrelation function of S(t). Again, there are certain restrictions on the moments of the marginal distributions as well as on the autocorrelation function.

Scalar Rectangular Wave Renewal Processes

Scalar rectangular wave renewal processes are useful models for processes changing their amplitude at random renewal points in a random fashion. A scalar rectangular wave renewal process **is** characterised by the jump rate λ, and the distribution function of the amplitude. The renewals occur independently of each other. No specific distribution is assigned to the interarrival times. Therefore, the renewal process characterised only by a jump rate captures only long term statistics. The mean duration of pulses is asymptotically equal to $1/\lambda$,. For the special case of a Poisson rectangular wave process the interarrival times and so the durations of the pulses are exponentially distributed with parameter $1/\lambda$.. In the special case of a Ferry Borges-Castanheta process the durations are constant and the repetition number $r = (t_2 - t_l)/\Delta$ with Δ the duration of pulses is equal to $\lambda(t_2 - t_1)$. Also, the sequence of amplitudes is an independent sequence.

The jump rate can be a function of time as well as the parameters of the distribution function of the amplitudes.

It is assumed that rectangular wave processes jump from a random value S(t) to a new value $S^+(t+\delta)$ with $\delta \rightarrow$ at a renewal without returning to zero. Rectangular wave renewal processes must be regular processes, i.e. the occurrence of any two or more renewals in a small time interval must be negligible (of o-order). Non-stationary rectangular wave renewal processes are processes which have either time-dependent parameters of the amplitude distributions and/or time-dependent jump rates.

Random fields

A random field may be regarded as a one-, two- or three-dimensional stochastic process. The time t is substituted by the space co-ordinates x, y, z.

For the two-dimensional case the covariance function is written (for a stationary random field)

$$r\,(d_x, d_y) = E\,[(F\,(x + d_x, y + d_y) - m)\,(F\,(x, y) - m)]$$

The concepts of stationary, ergodicy etc. are in principle the same as for the stochastic processes.

Vector processes

Two stationary Gaussian processes F1 and F2 are statistically completely described by their mean values, auto-spectra and the cross spectrum. The latter is defined by:

$$r_{12}(\tau) = E\,[(F_1(t + \tau) - m_1)(F_2(t) - m_2)]$$

$$S_{12}(n) = \int_{-\infty}^{\infty} e^{-i2\pi n\tau} r_{12}(\tau) d\tau$$

A vector of n stationary Gaussian processes can be described by n mean values, n auto-spectra and n(n-1) cross spectra. Note that S_{ij} is the complex conjugate of S_{ji}.

ANNEX 2 - DISTRIBUTIONS FUNCTIONS

Distribution type	Parameters	Moments
Rectangular $a \le x \le b$ $f_X(x) = \frac{1}{b-a}$	1 = a 2 = b	$m = \frac{a+b}{2}$ $s = \frac{b-a}{\sqrt{12}}$
Normal $\sigma > 0$ $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$	1 = μ 2 = σ	$m = \mu$ $s = \sigma$
Lognormal $x > 0, \zeta > 0$ $f_X(x) = \frac{1}{x\zeta\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln x - \lambda}{\zeta}\right)^2\right)$	1 = λ 2 = ζ	$m = \exp\left(\lambda + \frac{\zeta^2}{2}\right)$ $s = \exp\left(\lambda + \frac{\zeta^2}{2}\right)\sqrt{\exp(\zeta^2) - 1}$
Shifted Lognormal $x > \varepsilon, \zeta > 0$ $f_X(x) = \frac{1}{(x-\varepsilon)\zeta\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln(x-\varepsilon) - \lambda}{\zeta}\right)^2\right)$	1 = λ 2 = ζ 3 = ε	$m = \varepsilon + \exp\left(\lambda + \frac{\zeta^2}{2}\right) + \varepsilon$ $s = \exp\left(\lambda + \frac{\zeta^2}{2}\right)\sqrt{\exp(\zeta^2) - 1}$
Shifted Exponential $x \ge \varepsilon, \lambda > 0$ $f_X(x) = \lambda \exp(-\lambda(x-\varepsilon))$	1 = λ 2 = ε	$m = \frac{1}{\lambda} + \varepsilon$ $s = \frac{1}{\lambda}$
Shifted Gamma $x \ge 0, b > 0, p > 0$ $f_X(x) = \frac{b^p}{\Gamma(p)} \exp(-b(x-\varepsilon))(x-\varepsilon)^{p-1}$	1 = p 2 = b 3 = ε	$m = \frac{p}{b} + \varepsilon$ $s = \frac{\sqrt{p}}{b}$
Beta $a \le x \le b, \; r, t \ge 1$ $f_X(x) = \frac{(x-a)^{r-1}(b-x)^{t-1}}{(b-a)^{r+t-1} B(r,t)}$	1 = a 2 = b 3 = r 4 = t	$m = a + (b-a)\frac{r}{r+t}$ $s = \frac{b-a}{r+t}\sqrt{\frac{rt}{r+t+1}}$
Gumbel (Maximum) $-\infty < x < +\infty, \alpha > 0$ $f_X(x) = \alpha \exp(-\alpha(x-u) - \exp(-\alpha(x-u)))$	1 = u 2 = α	$m = u + \frac{0.577216}{\alpha}$ $s = \frac{\pi}{\alpha\sqrt{6}}$
Frechet (Maximum) $\varepsilon \le x < +\infty, \quad u, k > 0$ $f_X(x) = \frac{k}{u-\varepsilon}\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{-k-1} \exp\left(-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{-k}\right)$	1 = u 2 = k 3 = ε	$m = \varepsilon + (u-\varepsilon)\Gamma\left(1 - \frac{1}{k}\right)$ $s = (u-\varepsilon)\sqrt{\Gamma\left(1-\frac{2}{k}\right) - \Gamma^2\left(1-\frac{1}{k}\right)}$
Weibull (Maximum) $\varepsilon \le x < +\infty, \quad u, k > 0$ $f_X(x) = \frac{k}{u-\varepsilon}\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{-k-1} \exp\left(-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{-k}\right)$	1 = u 2 = k 3 = ε	$m = \varepsilon + (u-\varepsilon)\Gamma\left(1 + \frac{1}{k}\right)$ $s = (u-\varepsilon)\sqrt{\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)}$

ANNEX 3 MATHEMATICAL TECHNIQUES FOR LOAD COMBINATIONS

Combination of two rectangular processes (Ferry Borges-Castanheta model)

Consider the case that two actions $Q_1(t)$ and $Q_2(t)$ are to be combined. Assume that these actions can be described as rectangular or sqaure wave models (Figure A3.1). The following assumptions are made about the processes:

- Q_1 (t) and Q_2 (t) are stationary ergodic processes
- All intervals τ_1 are equal; all intervals τ_2 are equal and $\tau_1 \geq \tau_2$
- Q_1 and Q_2 are constant during each interval τ_1 and τ_2 respectively
- The values of Q_1 for the different intervals are mutually independent; same for Q_2
- Q_1 and Q_2 are independent

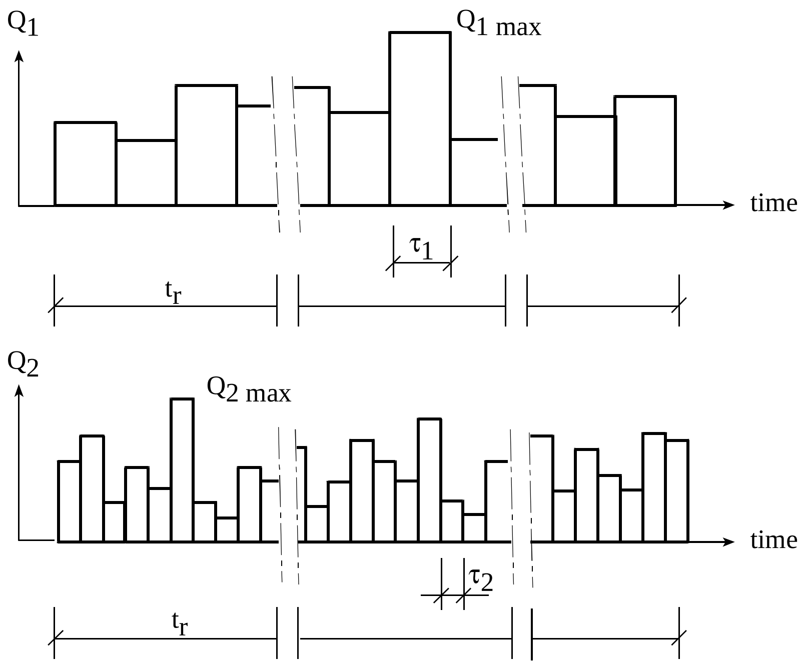

Figure A3.1: Square wave processes for Q_1 (t) and Q_2 (t)

Define Q_{2c} as the maximum value of Q_2 occurring during the interval τ_1 with the probability distribution function:

$$F_{Q2c}(Q) = \left[F_{Q^*}(Q)\right]^{\tau 2/\tau 1}$$

F_Q^* = the arbitrary point in time distribution for Q_2 (A3.1)

Assume a linear relationship between the actions effect E and the actions:

$$E = c_1 Q_1 + c_2 Q_2 \qquad \text{(A3.2)}$$

The maximum action effect E_{max} from Q_1 and Q_2 during the reference period t_o can then be written as:

$$E_{max} = \max \{c_1Q_1 + c_2Q_{2c}\} \qquad \text{(A3.3)}$$

The maximum should be taken over all intervals τ_1 within the reference period t_o.

As an approximation, the resulting action effects could be calculated as the maximum of the following two combinations (Turkstra's rule):

- $E\left\{Q_{1\max},\ Q_{2c}\right\}$ if Q_1 is considered as the dominating action
- $E\left\{Q_{2\max},\ Q_{1c}\right\}$ if Q_2 is considered as the dominating action

Written as a formula for the case $E = c_1\,Q_1 + c_2\,Q_2$

$$E_{max} = \max\left\{c_1Q_{max} + c_2Q_{2c}; c_1Q_{1c} + c_2Q_{2,max}\right\} \qquad \text{(A3.4)}$$

It should be noted that the Turkstra Rule gives a lower bound for the failure probability.

Oucrossing approach

Consider the more general event that random state vector $Z(\tau)$ representative for a given problem, enters the failure domain

$$V = \{Z(\tau) \mid g(z(\tau),\tau) < 0,\ 0 < \tau < t\};$$

where g(.) is the limit state function. $Z(\tau)$ may conveniently be separated into three components as:

$$Z(\tau)^T = (R^T, Q(\tau)^T, S(\tau)^T)$$

where R is a vector of random variables independent of time t, $Q(\tau)$ is a slowly varying random vector sequence and $S(\tau)$ is a vector of not necessarily stationary but sufficiently mixing random process variables having fast flunctuations as compared to Q(t).

In the general case where all the different types of random variables R, $Q(\tau)$ and $S(\tau)$ are present, the failure probability $P_f(t)$ not only must be integrated over the time invariant variables R, but an expectation operation must also be performed over the slowly varying variables $Q(\tau)$:

$$P_f(t_{min}, t_{max}) \approx 1 - E_R[\exp[nE_Q[1 - \exp(-E[N^+(\Delta t, R, Q))]]]] \qquad \text{(A3.5a)}$$

Δt is the characteristic fluctuation time of Q and $n = (t_{max} - t_{min}) / \Delta t$

Or, one step further simplified:

$$P_f(t_{min}, t_{max}) \approx 1 - E_R[\exp[-E_Q[E[N^+(t_{min}, t_{max}; R, Q)]]]] \qquad \text{(A3.5b)}$$

It should be observed that the expectation operation with respect to Q is performed inside the exponent, whereas the expectation operation with respect to R is performed outside the exponent operator. If the point process of exits is a regular process which can be assumed in most cases, the conditional expectation of the number of exits in the time interval $[t_{min}, t_{max}]$ can be determined from:

$$E[N^{+}(t_{min}, t_{max}; r, q)] = \int_{t_{min}}^{t_{max}} \nu^{+}(\tau; r, q) d\tau \qquad (A3.6)$$

where $\nu^{+}(\tau;p,r.q)$ is the outcrossing rate defined by:

$$\nu^{+}(\tau; r, q) = \lim_{\Delta \to 0} \frac{1}{\Delta} P(N^{+}(\{S(\tau) \in \bar{V}\} \cap \{S(\tau + \Delta) \in V\} | r, q) \qquad (A3.7)$$

If the vector $\underline{S}$ consists out of n components (S_1, ... S_n), all of ractangular wave type, the following formula can be used:

$$\nu^{+} = \sum_{i=1}^{n} \nu_i P\{(S_1, S_2, \ldots S_i, \ldots S_n) \in \bar{V}) \cap (S_1, S_2, \ldots S_i^{+}, \ldots S_n) \in V\} \qquad (A3.8)$$

where S_i^- and S_i^+ are two realisations of S_i, one before and one after some particular jump and ν_i is the jump rate of S_i.

Intermittent processes

Intermittent processes are a practically important generalisation for all types of random processes. Although more general forms are known only the simplest type of intermittancies is discussed below. The renewals of times where the process is "on" follow a Poisson renewal process with rate κ (or mean interarrival time l/κ). At a renewal the process activates an “on"-state (state "1"). The "off”-states are denoted by "0". The initial durations of "on"-states will have exponential distribution with mean l/μ independent of the arrival times. However, we will assume that a "on"-time is also finished if a next renewal occurs so that the durations have a truncated distribution. By assuming random initial conditions the probabilities of the “on/off'-states are then determined by

$$p_{off}(t) = \frac{\mu}{\kappa + \mu} + \frac{\kappa - \mu}{\kappa + \mu} \exp[-(\kappa + \mu)t] \qquad (A.3.9)$$

$$p_{on}(t) = \frac{\mu}{\kappa + \mu} + \frac{\kappa - \mu}{\kappa + \mu} \exp[-(\kappa + \mu)t] \qquad (A.3.10)$$

In general it is assumed that the "on/off”-process is already in its stationary state where the last terms in these equations vanishes. In contrast to rectangular wave renewal processes where the duration of the rectangular pulse is exactly until the next renewal and the duration of the rectangular pulse is exponentially distributed with mean l/λ for a Poissonian renewal process the "on"-times are now truncated at the next renewal. It is easily shown that the effective duration of the "on"-times then are also exponential but with mean $1/(\kappa+\mu)$. The so-called interarrival-duration intensity is defined by $\rho = \kappa/\mu$. For $\rho = \kappa/\mu \to \infty$ the processes are almost always active. For $\kappa/\mu \to 0$ one obtains spike-like processes.

Intermittancies can also be defined for differentiable processes. If this is a dependent vector process the entire vector process must have a common ρ, that is all components of the vector must have the same κ and μ. Independent differentiable vector processes, however, can have different ρ's.

In the case of a *single* intermittent process with $\kappa t_0 > 1$ and $\mu t_o << 1$ the periods where the intermittent load are present can conveniently be put together. The failure probability is then approximately given by:

$$P_f(t_{min}, t_{max}) = \nu_{on}\ T + \nu_{off}\ (t_o - T) \qquad \text{(A.3.11)}$$

where $T = \kappa\, t_0 / \mu$ = the total expected time that the intermittent load is active and $t_o = t_{max} - t_{min}$; ν_{on} and ν_{off} are the failure rates for present and absent intermittent load respectively.

In the case of two mostly absent uncorrelated intermittent loads, the same approximation principle can be applied, leading to:

$$\begin{aligned} P_f(t_{min}, t_{max}) = {} & (\frac{\kappa_1}{\mu_1})(\frac{\kappa_2}{\mu_2})\, \nu_{on,on}\, t_o + \\ & (\frac{\kappa_1}{\mu_1})(1-\frac{\kappa_2}{\mu_2})\, \nu_{on,off}\, t_o + \\ & (1-\frac{\kappa_1}{\mu_1})(\frac{\kappa_2}{\mu_2})\, \nu_{off,on}\, t_o + \\ & (1-\frac{\kappa_1}{\mu_1})(1-\frac{\kappa_2}{\mu_2})\, \nu_{off,off}\, t_o \end{aligned} \qquad \text{(A.3.12)}$$

where $\nu_{on,on}$ is the failure rate for both intermittent loads present, etc.

JCSS PROBABILISTIC MODEL CODE
PART 2 - LOAD MODELS

2.1 SELF WEIGHT

Table of contents:

List of symbols:

d = correlation length
V = volume described by the boundary of the structural part

γ = weight density of the material.
γ_{av} = average weight density for a structural part
ρ_o = correlation between two far away points in one member
Δr = distance between two points within a member

2.1.1 Introduction

The self weight concerns the weight of structural and non-structural components. The main characteristics of the self weight can be described as follows:

- The probability of occurrence at an arbitrary point-in-time is close to one
- The variability with time is normally negligible
- The uncertainties of the magnitude is normally small in comparison with other kinds of loads.

Concerning the uncertainties one can distinguish between (hierarchical model):

- variability **within** a structural part
- variability **between** different structural parts of the same structure
- variability **between** various structures

The variability within a structural part is normally small and can often be neglected. However, for some types of problem (c.g. static equilibrium) it may be important.

2.1.2 Basic model

The self weight, G, of a structural part is determined by the relation

$$G = \int_{Vol} \gamma \, dV \qquad (1)$$

where:

V is the volume described by the boundary of the structural part. The volume of V is Vol.
γ is the weight density of the material.

For a part where the material can be assumed to be reasonably homogeneous eq. (1) can be written

$$G = \gamma_{av} V \qquad (2)$$

where

γ_{av} is an average weight density for the part (see further section 2.1.4).

2.1.3 Probability density distribution functions

The weight density and the dimensions of a structural part are assumed to have Gaussian distributions. To simplify the calculations the self weight, G, may as an approximation be assumed to have a Gaussian distribution.

2.1.4 Weight density

Total variability

Mean values, μ_γ, and coefficients of variation, V_γ, for the total variability of the weight density of some common building materials are given in table 2.1.1.

Table 2.1.1. Mean value and coefficient of variation for weight density [1)]

Material	Mean value [kN/m^3]	Coefficient of variation
Steel	77	< 0.01
Concrete		
Ordinary concrete [2)]	24	0.04
High strength concrete	24-26 [4)]	0.03
Lightweight aggregate concrete	[4)]	0.04-0.08
Cellular concrete	[4)]	0.05-0.10
Heavy concrete for special purposes	[4)]	0.01-0.02
Masonry	-	≈ 0.05
Timber [3)]		
Spruce, fir (Picea)	4.4	0.10
Pine (Pinus)	5.1	0.10
Larch (Larix)	6.6	0.10
Beech (Fagus)	6.8	0.10
Oak (Quercus)	6.5	0.10

[1)] The values refer to large populations. They are based on data from various sources.
[2)] The values are valid for concrete without reinforcement and with stable moisture content. In case of continuous drying under elevated temperature the stable volume weight after 50 days is 1.0-1.5 kN/m^3 lower.
[3)] Moisture content 12%. An increase in moisture content from 12% to 22% causes a 10% rise in weight density.
[4)] Depends on mix, composition and treatment

Spatial correlations

Between densities of two points within one member, the following correlation can considered to be present:

$$\rho(\Delta r) = \rho_o + (1-\rho_o) \exp\{-(\Delta r/d)^2\} \qquad (3)$$

where

- d is a so called correlation length which characterises the correlation structures
- Δr is the distance between two points within a member
- ρ_o correlation between two far away points in one member

Only correlation in the length dimensions of a structural part are of importance. For beams the weight density over the cross section and for plates over the height may be considered as fully correlated.

Between points in two different members, but within one building, a constant correlation ρ_m is assumed to be present.

In the absence of more detailed information the following values can be used:

d	10 m (beam/column) 6 m (plate) 3 m (volume)
ρ_o	0.85
ρ_m	0.70

Note: For large members the variability of the weight density may be taken as V ρ_o; for a whole structure consisting out of many members the variability may be taken as V ρ_m , where V is the total variability according to table 2.1.1.

2.1.5 Volume

In most cases it may be assumed that the mean values of the dimensions are equal to the nominal values i.e. the dimensions given on drawings, in descriptions etc. The mean value of the volume, V, of the structural parts is calculated directly from the mean values of the dimensions.

The standard deviation of the volume, V, is calculated directly from the values of the standard deviation for the dimensions. Standard deviations for cross section dimensions are given in table 2.1.2 for some common building materials and types of structural elements.

Table 2.1.2. Mean values and standard deviations for deviations of cross-section dimensions from their nominal values.

Structure or structural member	Mean value	Standard deviation
Rolled steel		
steel profiles, area A	0.01 A_{nom}	0.04 A_{nom}
steel plates, thickness t	0.01 t_{nom}	0.02 t_{nom}
Concrete members [2)]		
$a_{nom} \leq 1000$ mm	0.003 a_{nom}	4 + 0.006 a_{nom}
$a_{nom} \geq 1000$ mm	3 mm	10 mm
Masonry members		
unplastered	0.02 a_{nom}	0.04 a_{nom}
plastered	0.02 a_{nom}	0.02 a_{nom}
Structural timber		
sawn beam or strut	0.05 a_{nom}	2 mm
laminated beam, planed	≈ 0	1 mm

1) The values refer to large populations. They are based on data from various sources and they concern members with currency acceptation dimension accuracy.
2) The values are valid for concrete members cast in situ. For concrete members produced in a factory the deviations may be considerably smaller.

The variability within a component **(**e.g. the variability of the cross section area along a beam) may be treated according to the same principles that is presented for the weight density in section 2.1.4.

Reference

CIB W81, Actions on Stuctures, Self weight, Report no. 115, Rotterdam

JCSS PROBABILISTIC MODEL CODE
PART 2: LOAD MODELS

2.2 LIVE LOAD

Table of contents:

List of symbols:

A = area [m^2]
d_p = duration of intermittend load [year]
i = influence function
m = mean load intensity in [kN/m^2]
p = intermittent load in [kN/m^2]
q = sustained load in [kN/m^2]
S = load effect in [kN/m^2]
T = reference time in [year]
V = zero mean normal distributed variable in [kN/m^2]
W = load intensity in [kN/m^2]

λ = occurrence rate of sustained load changes in [1/year]
ν = occurrence rate of intermittent load changes in [1/year]

2.2.1 Basic Model

The live loads on floors in buildings are caused by the weight of furniture, equipment, stored objects and persons. Not included in this type of load are any structural or non-structural elements, partition walls or extraordinary equipment. The live load is distinguished according the intended user category of the building, i. e. domestic buildings, hotels, hospitals, office buildings, schools, and stores. At design stage considerations should also be given to eventual changes of use during the life-time. Areas dedicated to store goods, materials, etc. must be treated separately. Live loads vary in time and space in a random manner. The spatial variations are assumed to be homogeneous in a first approximation. With respect to the variation in time, it is divided into two components, the sustained load and the intermittent load.

The sustained load contains the weight of furniture and heavy equipment. The load magnitude according to the model represents the time average of the real fluctuating load. Changes usually related to changes in use and of users in a building. Short term fluctuations are included in the uncertainties of this load part.

The intermittent load represents all kinds of live loads, which are not covered by the sustained load. The sources are like gathering of people, crowded rooms during special events, or stacking of furniture during remodelling. The relative duration of an intermittent loads is fairly small.

2.2.2 Stochastic Model

The load intensity is represented by a stochastic field $W(x,y)$, whereby the parameters depend on the user category of the building.

$$W(x,y) = m + V + U(x,y) \tag{1}$$

where m is the overall mean load intensity for a particular user category, V is a zero mean normal distributed variable and $U(x,y)$ is a zero mean random field with a characteristic skewness to the right. The quantities V and U are assumed to be stochastically independent.

The load effects calculated from the model shall describe the load effects caused by the real load, with sufficient accuracy. For linear elastic systems, where superposition is possible, the load effect S is written as:

$$S = \int_A W(x,y)\, i(x,y)\, dA \tag{2}$$

where $W(x,y)$ is the load intensity and $i(x,y)$ is the influence function for the load effect over a considered area A.

For non-linear structural response a stepwise linearity can be assumed, whereby the proposed relation for the load effect can be used in each step. The load intensity W is substituted by the step ΔW and the influence function $i(x,y)$ must reflect the total load situation, which results in a corresponding step ΔS for the load effect. When applying the theory of plasticity, then the influence function is proportional to the deflection corresponding to the mechanism.

An equivalent uniformly distributed load for the sustained load per unit area is that load having the same load effect as the original load field, i. e.

$$q = \frac{\int_A W(x,y) i(x,y) dA}{\int_A i(x,y) dA} \tag{3}$$

The statistical parameters of the sustained load are:

$$\begin{aligned} &E[q] = m \\ &Var[q] = \sigma_V^2 + \sigma_U^2 \frac{A_0}{A} \kappa \end{aligned} \tag{4}$$

whereby the factor κ is given in Figure (2.0.5.1) in Part 2.0. Note that for $A<A_0$ one should take $A_0/A = 1$.

The variable V describes the variability of sustained loads related to areas A_1 and A_2, which are assumed to be independent and non overlapping. These areas can be either on the same floor or on different floors. The covariance between the corresponding loads q_1 and q_2 is given as:

$$Cov[q_1, q_2] = \sigma_V^2 \tag{5}$$

The stochastic distribution of V is assumed to be normally distributed. The random field $U(x,y)$ has a specific skewness to the right, and in consequence also the load effect S and the sustained load q. A Gamma distribution for the sustained load fits best the actual observations, with parameters defined through the relations $E[q] = k/\mu_U$ and $Var[q] = k/\mu_U^2$.

The load intensity for the intermittent load p is represented by the same stochastic field as the sustained load, whereby the parameters depend on the user category of the building. The intermittent load can generally be considered as concentrated load. But, for design purposes, the same approach as for the sustained load is used. The duration of the intermittent load d_p is considered as deterministic.

The equivalent uniformly distributed load for intermittent loads p has the statistical properties as the sustained load and can be evaluated in the same manner. Generally, there is a lack of data for this load. The standard deviation normally gets values in the same magnitude as the mean value, $E[p] = \mu_p$. Therefore, the intermittent load is assumed to be exponentially distributed.

2.2.3 Variations in Time

The time between load changes is assumed to be exponentially distributed, then the number of load changes is Poisson distributed. The probability function for the maximum sustained load is given by:

$$F_{q_{max}}(x) = exp[-\lambda T(1 - F_q(x))] \tag{6}$$

where $F_q(x)$ is the probability function of the sustained load, T is the reference time, like the anticipated lifetime of the building, and λ is the occurrence rate of sustained load changes. Thus λT is the mean of the number of occupancy changes.

The maximum of the intermittent load is defined to occur as a Poisson process in time with the mean occurrence rate ν. The average duration of the intermittent load depends on the process, i.e. personnel, emergency or remodeling.

The maximum load which will occur in a building is a combination of sustained load and intermittent load. Assuming a stochastic independence between both load types, the maximum load during one occupancy is obtained from the convolution integral. The total maximum load during the reference time T is obtained by employing the extreme value theory.

In cases with high share in sustained load the duration statistics becomes of interest, especially for creep and shrinkage problems. Generally, the intermittent load will be of little interest. From the assumed extreme value distribution the statistical quantities of the excursion time τ over a certain level x can be derived.

$$\begin{aligned} E[\tau(x)] &= T(1 - F_q(x)) \\ Var[\tau(x)] &= 2T\,(1 - F_q(x))/\lambda \end{aligned} \tag{7}$$

2.2.4 Load Parameters

A list of parameters in table(2.2.1) are to be used in the live load model.

		Sustained Load				Intermittent Load			
Type of use	A_o [m²]	m_q [kN/m²]	σ_v [kN/m²]	σ_u [kN/m²]	$1/\lambda$ [a]	m_p [kN/m²]	σ_U [kN/m²]	$1/\nu$ [a]	d_P [d]
Office	20	0.5	0.3	0.6	5	0.2	0.4	0.3	1 - 3
Lobby	20	0.2	0.15	0.3	10	0.4	0.6	1.0	1 – 3
Residence	20	0.3	0.15	0.3	7	0.3	0.4	1.0	1 – 3
Hotel guest room	20	0.3	0.05	0.1	10	0.2	0.4	0.1	1 – 3
Patient room	20	0.4	0.3	0.6	5 – 10	0.2	0.4	1.0	1 – 3
Laboratory	20	0.7	0.4	0.8	5 – 10				
Libraries	20	1.7	0.5	1.0	>10				
School classroom	100	0.6	0.15	0.4	>10	0.5	1.4	0.3	1 – 5
Merchant/retail:									
first floor	100	0.9	0.6	1.6	1 – 5	0.4	1.1	1.0	1 – 14
upper floor	100	0.9	0.6	1.6	1 – 5	0.4	1.1	1.0	1 – 14
Storage	100	3.5	2.5	6.9	0.1–1.0				
Industrial:									
light	100	1.0	1.0	2.8	5 – 10				
heavy	100	3.0	1.5	4.1	5 – 10				
Concentration of people	20					1.25	2.5	0.02	0.5

Table 2.2.1 Parameters for live loads depending on the user category.

References

[1] CIB W81. Actions on Structures - Live Loads in Buildings. Conseil International du Bâtiment pour la Recherche l'Etude et la Documentation (CIB). Report 116, Rotterdam, 1989.
[2] EC 1-Part 2.1: Actions on structures - Densities, self-weight, imposed loads. Eurocode 1 - Basis of Design and Actions on Structures. Comité Européen de Normalisation (CEN). Pre-standard draft, Brussels, 1994.
[3] Rackwitz R: Live Loads in Buildings. Manuscript, unpublished, Munich, 1995.
[4] PMC Part 1: Basis of Design. Probabilistic Model Code - third draft. Joint Committee on Structural Safety (JCSS), 1995.

**JCSS PROBABILISTIC MODEL CODE,
PART 2, LOAD MODELS**

2.6 LOADS IN CAR PARKS

Table of contents:

List of symbols:

i = influence coefficient
t_d = busy time per year
t_y = busy days per year

L = weight of car in kN
S = load effect
T = reference time in years
N = number of parking places

λ_d = renewal rate in [1/d]
τ = mean dwell time in hours

2.6.1 Basic Model

In car parks the loads on parking areas and drive ways may be distinguished. In general, the loads for regulated parking are dominating the loads for spatially free parking. Further, the entries and parking places are such that only certain categories of vehicles can use the facility. It is sufficient to distinguish between facilities for light vehicles like normal passenger cars, station wagons and vans and for heavy vehicles like trucks and busses. For each parking facility it can conservatively be assumed that the vehicles form an independent sequence each vehicle with random weight remaining the same at arrival and when leaving the place. At the beginning of the busy periods it can conservatively be assumed that parking places left by a car will immediately be occupied by another car. Thus the loading process due to vehicles is a rectangular wave renewal process.

2.6.2 Stochastic Model

With respect to the temporal fluctuations one can distinguish the following usage categories for light vehicles:

- car parks belonging to residential areas
- car parks belonging to factories, offices etc.
- car parks belonging to commercial areas
- car parks belonging to assembly halls, sport facilities etc.
- car parks connected with railway stations airports etc.

The temporal fluctuations are summarized in table 1. For parking facilities for heavy vehicles similar distinctions can be made.

The mean weight of light vehicles can be assumed to be about $E[L] \approx 15$ kN with coefficient of variation of 15 to 30% depending on the usage of the parking facility and the traffic mixture. The parking place covers an area of about $2.4 \cdot 5.0$ m^2. A normal distribution can be assumed. In general, light vehicles can be modeled by point loads located in the middle of the parking places.

Location of car park	Busy days per year t_y[d]	Busy times per day t_d[h]	Mean dwell time τ[h]	Number of cars per day λ_d[1/d]
Commercial areas	312	8 4	2.4	3.2
Railway stations Airports	30	14-18	10-14	1.3
Assembly halls	50-150	2.5	2.5	1.0
Office factories	260	8-12	8-12	1.0
Residential areas	360	17	8	2.1

Table 1: Typical temporal fluctuations in car parks

Calculation of load effects has to take proper account of influence functions according to

$$S(t) = \sum_{j=i}^{n} i_j L_j \tag{1}$$

If the negative parts of the influence functions can be neglected the distribution of extreme load effects can be computed from

$$F_{\max\{S\}}(x) \approx \exp\left[-\lambda_d t_y t_d T P\left(\sum_{j=1}^{n} i_j L_j \geq x\right)\right] \quad (2)$$

with

$$P\left(\sum_{j=1}^{n} i_j L_j > x\right) \approx \Phi\left(-\frac{x - \left(\sum_{j=1}^{n} i_j E[L_j]\right)}{\left(\sum_{j=1}^{n} i_j Var[L_j]\right)^{1/2}}\right) \quad (3)$$

T is the reference time. On driveways where only one vehicle determines the load effect one has

$$F_{\max\{S\}}(x) \approx \exp\left[-\lambda_d t_y T N\left\{1 - \Phi\left(\frac{x - E[L]}{(Var[L])^{1/2}}\right)\right\}\right] \quad (4)$$

where N ist the numer of parking places associated with the drive way.

References

CIB W81, Actions on Structures: Live Load in Buildings, Rep. N0. 116, Rotterdam, 1989

JCSS PROBABILISTIC MODEL CODE
PART 2: LOAD MODELS

2.12 SNOW LOAD

Table of contents:

List of symbols:

C_e	= exposure coefficient
C_r	= redistribution (due to wind) coefficient
C_t	= deterministic thermal coefficient
d	= snow depth
h	= altitute of the building site
h_r	= reference altitude
k	= coefficient for altitude conversion
r	= conversion factor of snow load on ground to snow load on roofs
S_r	= snow load on the roof
S_g	= snow load on ground at the weather station
$\gamma(d)$	= average weight density of the snow for depth d
η_a	= shape coefficient

2.12 SNOW LOAD

2.12.1. Basic Model for Snow Load on roofs

The snow load on roofs, S_r , is determined by the relation

$$S_r = S_g \; r \; k^{h/h_r} \tag{1}$$

where

S_g is the snow load on ground at the weather station
r is a conversion factor of snow load on ground to snow load on roofs (see 2.12.3).
h is the altitute of the building site
h_r is a reference altitude (= 300 m)
k is a coefficient: k = 1.25 for coastal regions, k = 1.5 for inland mountainous regions

The snow load S_r acts vertically and refers to a horizontal projection of the area of the roof. S_g is time dependent but not space dependent within a specified region with similar climatic conditions and with approximately the same altitude.

The characteristics of the ground snow load S_g should be determined on the basis of observations from weather stations. The results of such observations are either water-equivalents of snow or depths of snow. In the first case the values can be used directly to determine the ground snow load. In the second case the data on snow depth must be converted to snow load by the relation

$$S_g = d \; \gamma(d) \tag{2}$$

where

d is the snow depth
$\gamma(d)$ is the average weight density of the snow

The density $\gamma(d)$ follows from:

$$\gamma(d) = \frac{\lambda\gamma(\infty)}{d} \ln\left\{1 + \frac{\gamma(0)}{\gamma(\infty)}[\exp(\frac{d}{\lambda}) - 1]\right\} \tag{3}$$

where

$\gamma(\infty)$ = 5 kN/m^3, $\gamma(0)$ = 1.70 kN/m^3 and x = 0.85 m

2.12.2. Probabilistic model for S_g

A probability model of the ground snow load S_g is presented by:

- a probability distribution function for the total duration T of the load
- a probability distribution function for the maximum load S_{gmax} within one year.

The distribution function $F_{sg\,max}$, its mean μ and its coefficient of variation V are denoted as:

for maritime climate : F_{s1} , μ_1 , V_1
for continental climate : F_{s2} , μ_2 , V_2

The probability distribution functions in these two cases are gamma distributions. The parameters should be based on local observations. As prior distribution a vague prior should be used. In some cases data from "similar stations" can be used as prior with n' = 3 and ν' = 2.

In those cases when the climate is a mixture of maritime and continental climate, a part p of the observations are associated with a continental climate and a part 1-p with a maritime climate. The combined probability distribution function F for the mixed climates can then be written as $F_s = (1-p)\,F_{s1} + p\,F_{s2}$.

2.12.3. Conversion ground to roof snow load

2.12.3.1 General

The conversion factor r is subdivided into a number of factors and terms according to the expression

$$r = \eta_a C_e C_t + C_r \tag{6}$$

where

η_a is a shape coefficient, a random variable according to 2.12.3.2
C_e is a deterministic exposure coefficient according to 2.12.3.2
C_t is a deterministic thermal coefficient according to 2.12.3.3
C_r is a redistribution (due to wind) coefficient, a random variable according to 2.12.3.4. If redistribution is not taken into account $C_r = 0$

2.12.3.2 The exposure coefficient C_e and shape factor η_a

The exposure coefficient, C_e and the shape factor η_a are a reduction coefficients taking account of the exposure to wind of a building and the slope of the roof α:

$\alpha = 0^o$	$C_e\eta_a = 0.4 + 0.6 \exp(-0{,}1\ u(H))$
$\alpha = 25^o$	$C_e\eta_a = 0.7 + 0.3 \exp(-0{,}1\ u(H))$
$\alpha = 60^o$	$C_e\eta_a = 0$

u(H) is the wind speed, averaged over a period of one week, at roof level H.
For intermediate values of α linear interpolation should be used.

2.12.3.3 The thermal coefficient C_t

The thermal coefficient, C_t , accounts for the reduction of snow load on roofs with high thermal transmittance, in particular glass covered roofs. C_t is equal to 1.0 for buildings which are not heated and for buildings where the roofs are highly insulated. A value of 0,8 shall be used for most other cases.

2.1.3.4 The redistribution coefficient C_r

The redistribution coefficient, C_r , takes account of the redistribution of the snow on the roof caused by wind but in some cases also by other causes.

For monopitch roofs the redistribution of snow load may be neglected.

For symmetrical duopitch roofs the coefficient C_r is assumed to be constant and equal to $\pm C_{ro}$ for each half of the roof according to FIG 1. C_{ro} has a β-distribution with $\mu(C_{ro})$ according to FIG 2; the coefficient of variation of C_r is equal to 1.0. For other types of roofs the numerical values given in ENV 1991-2-3 and ISO 4355 shall be used. These values can assumed to correspond to the mean value plus one standard deviation.

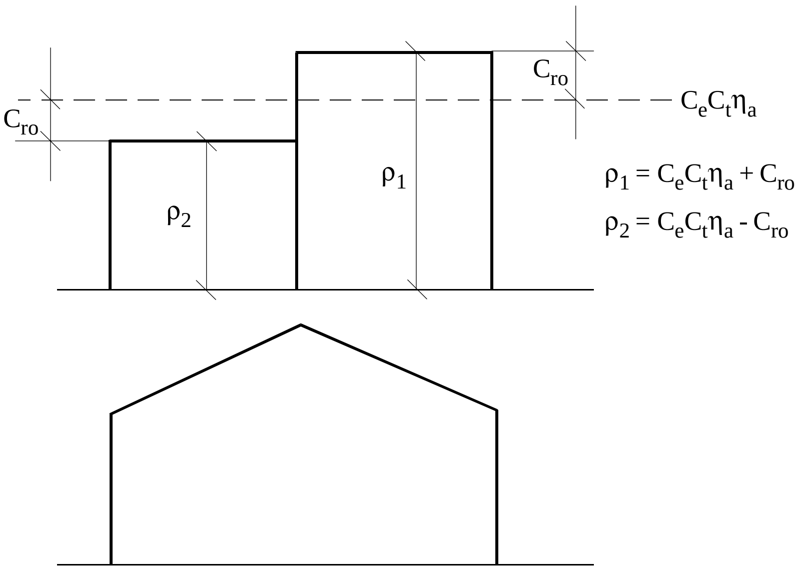

Figure 1: The redistributed snow load on a duopitch roof

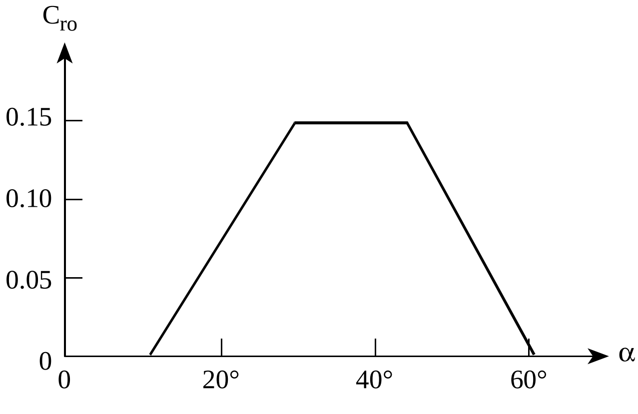

Figure 2: C_{ro} as function of the roof angle

Summary of snow load variables

X	designation	distribution	mean	scatter
S_g d_g	snow depth on the grund snowload on the ground	gamma	observation[1)]	observation[1)]
ρ	climate type parameter	det	observation	observation
k	parameter	det	1.5/1.25 m	-
h_r	reference height	det	300 m	-
$\gamma(0)$	unit weight at t = 0	det	1,7 kN/m^3	-
$\gamma(\infty)$	unit weight at t = ∞	det	5.0 kN/m^3	-
λ	parameter	det	0.85 m	-
$C_e\eta_a$	shape coefficient	beta	2.13.3.2	V = 0.15
C_t	insulation parameter	det	0.8-1.0	-
C_{ro}	redistribution coefficient	beta	Fig. 2	V = 1.0

[1)] Data from similar stations can be used as prior with n' = 3 and ν' = 2.

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PART 2 : LOADS

2.13 WIND

Table of contents:

List of symbols:

f_c	= Coriolis parameter (= $2\Omega \sin \phi$)
f_0	= mean frequency of zero up crossing, in Hz
g	= peak factor (no dimension)
$G_u(n)$, $G_v(n)$, $G_w(n)$	= half-sided power spectral density for longitudinal, transversal and vertical components of velocity fluctuations
$I_u(z)$	= turbulence intensity of longitudinal velocity fluctuations (dimensionless)
k	= von Karman`s constant (= 0.4)
$L_u(z)$	= integral length scale for longitudinal velocity fluctuations, in m
$L_v(z)$	= integral length scale for transversal velocity fluctuations, in m
$L_w(z)$	= integral length scale for vertical velocity fluctuations, in m
N	= number of reference time, in years
n	= frequency, in Hertz
n_u, n_v, n_w	= dimensionless frequency of fluctuations in longitudinal, transversal and vertical direction
$\overline{Q}_{ref}$	= reference wind velocity pressure

List of symbols:

$\overline{Q}(z)$	= mean velocity pressure at height z (=(1/2) ρ $\overline{U}^2(z)$)
$S_{ij}(n)$	= cross spectral power density
T	= reference time
$\overline{T}(u_p)$	= mean recurrence interval of maximum annual mean velocity, in years
$\overline{U}_{ref}$	= reference wind velocity, in m/s
$\overline{U}(z)$	= mean longitudinal velocity of the wind at height z
u_1	= mode of the maximum annual mean wind speed in Gumbel distribution
u(x,z,t)=u	= longitudinal component of the wind velocity fluctuations, in m/s
v(y,z,t)=v	= transversal component of wind velocity fluctuations, in m/s
w(z,t)=w	= vertical component of wind velocity fluctuations, in m/s
z	= height above ground, in m
z_0	= roughness length, in m
z_r	= a reference height above ground, in m
z_{ref}	= the reference height above ground (10 - 30 m)
α_1	= dispersion parameter for the maximum annual mean wind speed in Gumbel distribution
δ	= height of the atmospheric boundary layer
κ	= surface drag coefficient (dimensionless) (=$[k/\ln(z_{ref}/z_0)]^2$)
λ_k	= k-th moment of spectral density
ν(x)	= mean upcrossing rate for level x
φ	= geographical latitude
ρ	= air density (= 1.25 kg/m^3)
σ_u, σ_v,σ_w	= standard deviation of velocity fluctuations in x-, y- and z-direction, in m/s

2.13.1 Introduction

Wind effects on buildings and structures depend on the general wind climate, the exposure of buildings, structures and their elements to the natural wind, the dynamic properties, the shape and dimensions of the building (structure). The section presents basic data and procedures for the estimation of wind loads on buildings and structures. Tropical cyclones, tornados, thunderstorms and orographic wind phenomena require separate treatment.

The field of wind velocities over horizontal terrain is decomposed into a mean wind (average over 10 minutes) in the direction of general air flow (x-direction) averaged over a specified time interval and a fluctuating, turbulent part with zero mean and components in the longitudinal (x) direction, the transversal (y-) direction and the vertical (z-) direction

2.13.2 Wind forces

The wind force acting per unit area of structure is determined with the relations:

(i) For rigid structures of smaller dimensions:

$$w = c_a c_g c_r \overline{Q}_{ref} = c_a c_e \overline{Q}_{ref} \qquad (1)$$

(ii) For structures sensitive to dynamic effects (natural frequency < 1Hz) and for large rigid structures:

$$w = c_d c_a c_e \overline{Q}_{ref} \qquad (2)$$

where:

$\overline{Q}_{ref}$ = the reference (mean) velocity pressure
c_r = roughness factor
c_g = gust factor
c_a = aerodynamic shape factor
c_d = dynamic factor.

2.13.3 Mean wind velocity

The reference wind velocity, $\overline{U}_{ref}$ is the mean velocity of the wind averaged over a time interval of 10 min = 600 s, determined at an elevation of 10 m above ground, in horizontal open terrain exposure (z_0 = 0.03 m).[1]

The distribution of the mean wind velocities (for any terrain category, height above ground and averaging time interval) is the Weibull distribution:

$$F_{\overline{U}}(x) = 1 - \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma}\right)^k \right] \qquad (3)$$

with k close to 2.

[1] For other than 10 min averaging intervals, in open terrain exposure, the following relationships may be used: $1.05\overline{U}^{1h} = 1.0\overline{U}^{10min} = 0.84\overline{U}^{1min(fastest\ mile)} = 0.67\overline{U}^{3sec}$.

The same distribution is valid for direction dependent mean wind flows. Generally, it can not be assumed that the mean wind direction is uniformly distributed over the circle.

Mean wind velocities vary over the year. If no data are available it can be assumed in the northern hemisphere that $\sigma(t) \approx \sigma[1+ a \cos(2\pi(t-t_0)/365]$ with the constant a between 1/3 and 1/2 and $t_0 \approx 15$ to 45, with t in days.

The mean wind velocities are highly autocorrelated. Mean wind velocities with separation of about 4 to 12 (8 on average) hours can be considered as independent in most practical applications.

If seasonal variations are neglected, the mean period the mean wind velocities are between levels x_1 and x_2 $(x_2 \geq x_1)$ is asymptotically

$$E\left[T_{x_1,x_2}\right] = T[F_{\bar{U}}(x_2) - F_{\bar{U}}(x_1)] \tag{4}$$

with T the reference time. For higher levels of x_2 the distribution of individual times above x is approximately $[1 - F_{\bar{U}}(x)]/\nu(x)$ with $\nu(x)$ the mean upcrossing rate for level x.

The maximum mean wind speeds for longer periods follows a Gumbel distribution for maxima. Generally, it is not possible to infer the maxima over more years from observations covering only a few years. If the annual maxima are used, provided that the maximum annual data are homogenous as exposure and averaging time, the distribution function is:

$$F_{\max \bar{U}}(x) = \exp\{-\exp[-\alpha_1(x - u_1)]\} \tag{5}$$

The mode u and the parameter α_1 of the distribution are determined from the mean m_1 and the standard deviation σ_1 of the set of maximum annual velocities: $u_1 = m_1 - 0.577/\alpha_1$, $\alpha_1 = 1.282/\sigma_1$. The coefficient of variation of maximum annual wind speed, $V_1 = \sigma_1 / m_1$ depends on the climate and is normally between 0.10 and 0.35. For reliable results, the number of the years of available records must be of the same order of magnitude like the required mean recurrence interval.

The lifetime (N years) maxima of wind velocity is also Gumbel distributed and the mean and the standard deviation of lifetime maxima are functions of the mean and of the standard deviation of annual maxima: $m_N + 0.78\,\sigma_1 \ln N$, $\sigma_N = \sigma_1$.

Under special climatic conditions, the distribution of mean wind speeds is a mixed distribution reflecting different meteorological phenomena.

For load combination purposes it is proposed to model storms, for example those wind regimes where a mean velocity > 10 m/s lasts for some time, as an intermittent rectangular wave renewal process. The number of storms per year is approximately 50 corresponding to the frequency with which weather systems pass by, at least in middle Europe. The mean duration of the storm is approximately 8 hours. Consecutive storms are independent. The representative mean wind velocity in a storm can also be modeled by a Weibull distribution. The exponent of the Weibull distribution should be around 2. The location parameter should be based on local data.

2.13.4 Terrain roughness (category)

The roughness of the ground surface is aerodynamically described by the roughness length z_0, which is a measure of the size and spacing of obstacles on the ground surface. Alternatively, the terrain roughness can be described by the surface drag coefficient, κ corresponding to the roughness length z_0:

$$\kappa^2 = \frac{k}{\ln\frac{z_{ref}}{z_0}} \tag{6}$$

where $k \cong 0.4$ is von Karman´s constant and z_{ref} is the reference height (Table 2, Table 3). Various terrain categories are classified in Table 1 according to their approximate roughness lengths. The distribution of the surface roughness with wind direction must be considered.

Table 1. Roughness length z_0, in meters, for various terrain categories [1) 2)]

Terrain category	Terrain description	Range of z_0	Recom-mended value
A. Open sea. Smooth flat country	Areas exposed to the wind coming from large bodies of water; snow surface; Smooth flat terrain with cut grass and rare obstacles.	0.0001 | 0.0005	0.003
B. Open country	High grass (60 cm) hedges, and farmland with isolated trees; Terrain with occasional obstructions having heights less than 10 m (some trees and some buildings)	0.01 | 0.1	0.03
C. Sparsely built-up urban areas. Wooded areas	Sparsely built-up areas, suburbs, fairly wooded areas (many trees)	0.1 | 0.7	0.3
D. Densely built-up urban areas. Forests	Dense forests in which the mean height of trees is about 15m; Densely built-up urban areas; towns in which at least 15% of the surface is covered with buildings having heights over 15m	0.7 | 1.2	1.0
E. Centers of very large cities	Numerous large high closely spaced obstructions: more than 50% of the buildings have a height over 20m	1.0 ≥ 2.0	2.0

[1)] Smaller values of z_0 provoke higher mean velocities of the wind
[2)] For the full development of the roughness category, the terrains of types A to D must prevail in the up wind direction for a distance of at least of 1000m, respectively. For category E this distance is more than 5 km.

2.13.5 Variation of the mean wind with height

The variation of the mean wind velocity with height over horizontal terrain of homogenous roughness can be described by the logarithmic law. The logarithmic profile is valid for moderate and strong winds (mean hourly velocity > 10 m/s) in neutral atmosphere (where the vertical thermal convection of the air may be neglected).

$$\overline{U}(z) = \frac{1}{k} u_*(z_0)\left(\ln\frac{z}{z_0} + 5.75\frac{z}{\delta} - 1.87\left(\frac{z}{\delta}\right)^2 - 1.33\left(\frac{z}{\delta}\right)^3 + 0.25\left(\frac{z}{\delta}\right)^4 \right) \quad (z > d_0 >> z_0) \tag{7}$$

where:

u(z0) $= \dfrac{\overline{U}(z)}{2.5\,ln\dfrac{z}{z_0}}$ = friction velocity in m/s

δ $= \dfrac{u_*(z_0)}{6 f_c}$ = depth of boundary layer in m

$\overline{U}(z)$ = mean velocity of the wind at height z above ground in m/s
z = height above ground in m
z_0 = roughness length in m
k = von Karman's constant (k ≅ 0.4
d_0 = the lowest height of validity of Eq.(7) in m
f_c = 2Ωsin(ϕ) = Coriolis parameter in 1/s
Ω = 0.726 10^{-4} = angular rotation velocity in rad/s
ϕ = latitude of location in degree

For lowest 0.1 δ or 200m of the boundary layer only the first term needs to be taken into account (Harris and Deaves, 1981). The lowest height of validity for Eq.(7), d_0, is close to the average height of dominant roughness elements : i.e. from less than 1 m, for smooth flat country to more than 15 m, for centers of cities. For $z_0 \le z \le d_0$ a linear interpolation is recommended. In engineering practice, Eq.(7) is conservatively used with $d_0 = 0$.

With respect to the reference (open terrain) exposure, the relation between wind velocities in two different roughness categories at two different heights can be written approximately as (Bietry, 1976, Simiu, 1986):

$$\frac{\overline{U}(z)}{\overline{U}_{ref}} = \frac{\ln\dfrac{z}{z_0}}{\ln\dfrac{z_{ref}}{z_{0,ref}}}\left(\frac{z_0}{z_{0,ref}}\right)^{0.07} . \qquad (8)$$

At the reference height z_{ref}, the ratio of the mean wind velocity in various terrain categories to the mean wind velocity in open terrain is given by the factor p in Table 2. The corresponding ratio for the mean velocity pressure is p^2 .

Table 2. Scale factors for the mean velocity (and the mean velocity pressure) at reference height in various terrain exposure

Terrain category	A. Open sea. Smooth flat country	B. Open country	C. Sparsely built-up urban areas. Wooded areas	D. Densely built-up urban areas. Forests	E. Centers of large cities
z_{ref}, m	10	10	10	15	30
p	1.19	1.00	0.71	0.56	0.39

2.13.6 Intensity of turbulence

The turbulent fluctuations of the wind velocity can be assumed to be normally distributed with mean zero. The root mean squared value of the velocity fluctuations in the airflow, deviating from the longitudinal mean velocity, may be normalised to the friction velocity as follows:

$$\frac{\sigma_u}{u_*} = \beta_u \left(1 - \frac{z}{\delta}\right)$$ Longitudinal (9a)

$$\frac{\sigma_v}{u_*} = \beta_v \left(1 - \frac{z}{\delta}\right)$$ Transversal (9b)

$$\frac{\sigma_w}{u_*} = \beta_w \left(1 - \frac{z}{\delta}\right)$$ Vertical . (9c)

The approximate linear variation with height (Hanna, 1982) can be used only in moderate and strong winds. For neutral atmosphere, the ratios σ_v/σ_u and σ_w/σ_u near the ground are constant irrespective the roughness of the terrain (ESDU 1993):

$$\frac{\sigma_v}{\sigma_u} = 1 - 0.25\cos^4\left(\frac{\pi}{2}\frac{z}{\delta}\right) \quad (10a)$$

$$\frac{\sigma_w}{\sigma_u} = 1 - 0.55\cos^4\left(\frac{\pi}{2}\frac{z}{\delta}\right) \quad (10b)$$

For z<<δ the variance of the velocity fluctuations can be assumed independent of height above ground :

$$\sigma_u = \beta_u u_* \quad (11a)$$

$$\sigma_v = \beta_v u_* \quad (11b)$$

$$\sigma_w = \beta_w u_* \quad (11c)$$

and, for z < 0.1 δ:

$$\frac{\sigma_v}{\sigma_u} \cong 0.75 \quad (12a)$$

$$\frac{\sigma_w}{\sigma_u} \cong 0.50 \quad (12b)$$

The variance of the longitudinal velocity fluctuations can also be expressed from non-linear regression of measurement data, as function of terrain roughness (Solari, 1987):

$$4.5 \le \beta_u^2 = 4.5 - 0.856\ln z_0 \le 7.5 \quad (13)$$

The longitudinal intensity of turbulence is the ratio of the root mean squared value of the longitudinal velocity fluctuations to the mean wind velocity at height z (i.e. the coefficient of variation of the velocity fluctuations at height z :

$$I_u(z) = \frac{\overline{u(z,t)^2}^{1/2}}{\overline{U}(z)} = \frac{\sigma_u(z)}{\overline{U}(z)} \quad (14)$$

The turbulence intensity at height z can be approximated by:

$$I(z) = \frac{\beta_u}{2.5 \ln \frac{z}{z_0}} \approx \frac{1}{\ln \frac{z}{z_0}} \qquad (15)$$

The transversal and vertical intensities of turbulence can be determined by multiplication of the longitudinal intensity $I_u(z)$ by the ratios σ_v/σ_u and σ_w/σ_u. Representative values for intensity of turbulence at the reference height are given in Table 3.

Table 3: Wind parameters depending on terrain category

Terrain category	A. Open sea. Smooth flat country	B.Open country	C. Sparsely built-up urban areas. Wooded areas	D. Densely built-up urban areas. Forests	E. Centers of large cities
z_0 [m]	0.01	0.05	0.3	1.0	2.0
d_0 [m]	0	2	8	15	30
κ	0.0024	0.0047	0.013	0.022	0.022
β_u	3.1	2.7	2.3	2.1	2.0
β_v	2.3	2.1	1.8	1.6	1.5
β_w	1.55	1.35	1.15	1.05	1.0
z_{ref} [m]	10	10	10	15	30
$I(z_{ref})$	0.15	0.19	0.26	0.31	0.39

2.13.7 Power spectral density and autocorrelation functions of gustiness

The normalised half-sided von Karman power spectral densities and autocorrelation functions of gust velocity are given in Table 4.

Table 4. The von Karman model of isotropic turbulence

Component of gust velocity	Normalised spectral density $\frac{nG_i(n)}{\sigma_i^2}$	Normalised autocorrelation function $\rho_i(\tau_i)$
Longitudinal I = u	$\frac{4n_u}{\left(1+70.8\,n_u^2\right)^{5/6}}$	$\frac{2^{2/3}}{\Gamma(1/3)}\bar{\tau}_u^{1/3}\,K_{1/3}\left(\bar{\tau}_u\right)$
Transversal I = v Vertical i = w	$\frac{2n_i\left(1+188.6\,n_i^2\right)}{\left(1+70.8\,n_i^2\right)^{11/6}}$	$\frac{2^{2/3}}{\Gamma(1/3)}\bar{\tau}_i^{1/3}\left[K_{1/3}(\bar{\tau}_i)-\frac{1}{2}\bar{\tau}_i K_{2/3}(\bar{\tau}_i)\right]$

The notations in Table 4 are as follows:

σ_i^2 = variance of velocity fluctuations in direction i, in m^2/s^2; i = u, v or w

n_i = $n_i(z) = \frac{n\,L_i(z)}{\bar{U}(z)}$ = is a non-dimensional height dependent frequency

n = frequency, in Hertz

$\overline{U}(z)$	= longitudinal mean velocity at height z, in m/s
$L_i(z)$	= length of integral scale of turbulence in direction i, in m/s.
$\overline{\tau}_i = \dfrac{\tau\overline{U}(z)}{aL_i(z)}$	= non-dimensional time (a = 1.339)
$K_\mu(\ )$	= modified Bessel function of second kind of order μ
τ	= time lag, in s

The integral length scale of turbulence in direction i at the height z is:

$$L_i(z) = U(z)\int_0^\infty \rho_i(\overline{\tau}_i)\,d\tau_i \qquad (16)$$

where the autocorrelation $\rho_i(\tau_i)$ is the Fourier transform of spectral density. An estimation of the length of the integral scale of longitudinal turbulence, for heights up to 300 m is given by ESDU (1993), as:

$$L_u(z) = \frac{A^{3/2}(\sigma_u/u_*)^3 z}{2.5K_z^{3/2}(1-z/h)^2(1+5.75z/h)} \qquad (17)$$

where

$$A = 0.115\left[1+0.315\left(1-\frac{z}{\delta}\right)^6\right]^{2/3}$$

$$K_z = 0.188[1-(1-z/z_c)^2]^{1/2}$$

$$z_c/\delta = 0.39\left[\frac{u_*}{f_c z_0}\right]^{-1/8}$$

For the lateral and vertical direction (ESDU, 1993):

$$L_v(z) = 0.5\,(\sigma_v/\sigma_u)^3\,L_u(z) \qquad (18a)$$
$$L_w(z) = 0.5\,(\sigma_w/\sigma_u)^3\,L_u(z) \qquad (18b)$$
$$L_v(z) \cong 0.24\,L_u(z) \qquad (18c)$$
$$L_w(z) \cong 0.08\,L_u(z) \qquad (18d)$$

2.13.8 Coherence functions

The cross-spectral density for two separated points P_1 and P_2 with distance r perpendicular to direction i are given in terms of the point spectra and the coherence function by:

$$S_{ij}(n,P_1,P_2) \approx S_{ii}^{1/2}(n,P_1,P_2)S_{jj}^{1/2}(n,P_1,P_2)\cdot Coh_{ij}^{1/2}(n,P_1,P_2) \qquad (19)$$

with:

Longitudinal $$Coh_{uu}^{1/2}(r,\overline{k}) = \frac{\left(\dfrac{\psi_u}{2}\right)^{5/6}}{\Gamma(5/6)}\left[2K_{5/6}(\psi_u)-\psi_u K_{1/6}(\psi_u)\right] \approx exp(-1.15\psi_u^{1.5} \qquad (20a)$$

Transversal $$Coh_{vv}^{1/2}(r,\bar{k}) = \frac{\left(\frac{\psi_v}{2}\right)^{5/6}}{\Gamma(5/6)}\left[2K_{5/6}(\psi_v) + \frac{6(r\bar{k})^2}{3\psi_v^2 + 5(r\bar{k})^2}\psi_v K_{1/6}(\psi_v)\right] \approx exp(-0.65\psi_v^{1.3})$$

(20b)

Vertical $$Coh_{ww}^{1/2}(r,\bar{k}) = \frac{\left(\frac{\psi_w}{2}\right)^{5/6}}{\Gamma(5/6)}\left[2K_{5/6}(\psi_w) - \frac{6(rL)^2}{3\psi_w^2 + 5(r\bar{k})^2}\psi_w K_{1/6}(\psi_w)\right] \approx exp(-0.65\psi_w^{1.3})$$

(20c)

where $\bar{k} = \frac{2\pi n}{\bar{U}_m}$ and $\psi_i^2 = \left(r^2\bar{k}^2 + r^2 / L_i^2\right)$. All coherence function $Coh_{ij}^{1/2}(n, P_1, P_2)$ with i≠j can be assumed to vanish.

The longitudinal coherence can also be approximated by (Kareem, 1987):

$$Coh_{uu}^{1/2}(n,r) \approx \exp\left[-\left\{\left(\frac{r}{L_u}\right)^2 + \left(\frac{nr}{\bar{U}_m}\right)^2\left(12 + \frac{11r}{z_m}\right)^2\right\}^{1/2}\right] \tag{21}$$

implying a coherence coefficient of $C = 12 + 11r / z_m$ and where

$$z_m = \sqrt{z_1 z_2}$$
$$\bar{U}_m = \sqrt{\bar{U}_1(z_1)\bar{U}_2(z_2)}\,.$$

For structures of small dimension, i.e. r much smaller than L_u, r can be taken as zero.

2.13.9 Peak velocities

Spectral moments, λ_i of higher than the i = 0 order formally do not exist for turbulence spectra (including von Karman and other spectra) fulfilling the Kolmogorov asymptote (asymptotic $f^{-5/3}$ behaviour). However, for high frequencies the spectra fall off more rapidly so that truncation of these spectra at frequencies of 5÷20 Hz makes them finite. Also, filtering by finite areas on which the wind blows removes this mathematical inconvenience. Then, the distribution of extreme gust velocities, u_{max} is asymptotically a Gumbel distribution with mean:

$$E\left[u_{max} \middle| \lambda_0, \lambda_2, t\right] = \left(\sqrt{2\ln \nu_0 t} + \gamma / \sqrt{2\ln \nu_0 t}\right)\sqrt{\lambda_0} \tag{22}$$

and variance:

$$Var\left[u_{max} \middle| \lambda_0, \lambda_2, t\right] = [(\pi^2 / 6) / 2\ln \nu_0 t]\lambda_0 \tag{23}$$

γ=0.5772 is Euler´s constant, t = 600 s and ν_0 is the mean frequency of zero upcrossing, in Hz:

$$\nu_0 = \sqrt{\lambda_2 / \lambda_0}\ . \tag{24}$$

The mean and standard deviation of the random peak factor for gust velocities, g are defined as:

$$g = \sqrt{2\ln \nu_0 t} + 0.577 / \sqrt{2 \ln \nu_0 t} \tag{25}$$

$$\sigma_g = \frac{\pi}{6} \frac{1}{\sqrt{2 \ln \nu_0 t}} \tag{26}$$

The calculation of g from turbulence spectra is sensitive to the choice of cut-off frequency (5-20 Hz). Empirically and theoretically one can assume that the mean of g is about 3.2 for 1 hour (3.8 for 8 hours) and its standard deviation about 0.4. Since the fluctuating velocity pressure is a linear function of fluctuating velocity of gusts, the above values of g and σ_g also apply to the peak pressure.

2.13.10 Mean velocity pressure and exposure factor

The mean wind velocity pressure [2]) at height z is defined by:

$$\overline{Q}(z) = \frac{1}{2} \rho \overline{U}^2 (z) \tag{27}$$

where ρ is the air density (ρ=1.25 kg/m^3 for standard air).

The coefficient of variation of the maximum annual velocity pressure is approximately the double of the coefficient of variation of the maximum annual velocity, V_1 : $V_Q \cong 2\ V_1$.

The roughness factor describes the variation of the mean velocity pressure with height above ground and terrain roughness as function of the reference velocity pressure. From Eq.(13) one gets:

$$c_r(z) = \frac{\overline{Q}(z)}{\overline{Q}_{ref}} = \frac{\overline{U}(z)^2}{\overline{U}^2_{ref}} = \left[\frac{\left(\frac{z}{z_{0,ref}} \right)^{0.07}}{\ln \frac{z_{ref}}{z_{0,ref}}} \right]^2 \left(\ln \frac{z}{z_0} \right)^2 \tag{28}$$

and $\overline{Q}(z) = c_r(z)\ \overline{Q}_{ref}$ (29)

2.13.11 Gust factors for velocity pressure

[2] Conversion of the open country velocity pressure for different averaging time intervals can be guided by the following values obtained from Section 2.13.2:

$$1.1\overline{Q}^{1h} = \overline{Q}^{10min} = 0.7\overline{Q}^{1min(fastest\ mile)} = 0.44\overline{Q}^{3s}$$

The gust factor for velocity pressure is the ratio of the peak velocity pressure to the mean velocity pressure of the wind:

$$c_g(z) = \frac{q_{peak}(z)}{\overline{Q}(z)} = \frac{\overline{Q}(z) + g \cdot \sigma_q}{\overline{Q}(z)} = 1 + g \cdot V_q = 1 + g\left[2 \cdot I_u(z)\right] \qquad (29)$$

where:

$\overline{Q}(z)$ = the mean velocity pressure of the wind

σq = $\overline{q(z,t)^2}^{1/2}$ = the root mean squared value of the longitudinal velocity pressure fluctuations from the mean

V_Q = coefficient of variation of the velocity pressure fluctuations (approximately equal to the double of the coefficient of variation of the velocity fluctuations):

V_Q ≅ 2 I(z)

g = the peak factor for velocity pressure.

Approximately, the longitudinal velocity pressure fluctuation, q(z,t) is a linear function of the velocity fluctuation. Since:

$$\frac{1}{2}\rho\left[\overline{U}(z)^2 + u(z,t)\right]^2 = \frac{1}{2}\rho\overline{U}(z)^2 + \rho\overline{U}(z)u(z,t) + \frac{1}{2}\rho u(z,t)^2 \cong \frac{1}{2}\rho\overline{U}(z)^2 + \rho\overline{U}(z)u(z,t)$$

it is:

$$\overline{Q}(z) = \frac{1}{2}\rho\overline{U}(z)^2$$

$$q(z,t) \cong \rho\overline{U}(z)u(z,t)$$

and consequently, the mean value and the standard deviation of the peak factor for 10 min. velocity pressure are the same like that for the gust velocity g ≅ 3.2 and σ_g ≅ 0.4. The values of the peak factor depend on the averaging time interval of the reference velocity.[3]

2.13.12 Exposure factor for peak velocity pressure

The peak velocity pressure at the height z above ground is the product of the gust factor: the roughness factor and the reference velocity pressure;

$$Q_g(z) = c_g(z)\, c_r(z)\, \overline{Q}_{ref} \qquad (30)$$

The exposure factor is defined as the product of the gust and roughness factors:

$$c_e(z) = c_g(z)\, c_r(z). \qquad (31)$$

[3] Since: $q_{peak} = c_g^{1\,min}\left(c_r Q_{ref}^{1min}\right) = c_g^{10\,min}\left(c_r Q_{ref}^{10min}\right) = c_g^{1\,h}\left(c_r Q_{ref}^{1h}\right)$ from Section 2.13.8, the following approximate relations hold: $c_g^{1\,min} = 0.7 c_g^{10\,min} = c_g^{1h}$

2.13.13 Aerodynamic shape factors

The aerodynamic shape factor, c_a is the ratio of the aerodynamic pressure exerted by the wind on the surface of a structure and its components to the velocity pressure. The aerodynamic pressure is acting normal to the surface. By convention c_a is assumed positive for pressures and negative for suctions.

As the pressure exerted on a surface is not uniformly distributed over the whole area of the surface or on the different faces of a building, the aerodynamic coefficients should be assessed separately for the different parts and faces of a building.

The aerodynamic shape factors refer either to the mean pressure or to the peak pressure of the wind.

The shape factors are dependent on the geometry and the dimensions of building, the angle of attack of the wind i.e. the relative position of the body in the airflow, terrain category, Reynolds number, etc.

In certain cases the aerodynamic factors for external pressure must be combined with those for internal pressure.

There are two different approaches to the practical assessment of wind effects on rigid structures: using pressure coefficients and using force coefficients.

- In the former case the wind force is the result of the summation of the aerodynamic forces normal to a certain surface. It is intended for parts of the structure.
- In the later case, the wind force is the product of the velocity pressure multiplied by the overall force coefficient times the frontal area of the building. This approach is used within the procedures for calculating the structural response.

Typical values of the aerodynamic shape factors can be selected from appropriate national and international documents or from wind tunnel tests. The aerodynamic shape factors should be determined in wind tunnels capable of modelling the atmospheric boundary layer.

2.13.14 Uncertainties consideration

The factors involved in the assessment of the wind forces on structures contain uncertainties.

The mean and the coefficient of variation of the wind forces expressed through the product of uncorrelated variables in Eq.(1) or Eq.(2) may be written as follows:

$$E(w) = E(c_g)\, E(c_a)\, E(c_r)\, E(\overline{Q}_{ref}) \tag{32}$$

$$V_w^2 = V_{c_g}^2 + V_{c_a}^2 + V_{c_r}^2 + V_{\overline{Q}_{ref}}^2 \tag{33}$$

and

$$E(w) = E(c_d)\, E(c_a)\, E(c_r)\, E(\overline{Q}_{ref}) \tag{34}$$

$$V_w^2 = V_{c_d}^2 + V_{c_a}^2 + V_{c_r}^2 + V_{\overline{Q}_{ref}}^2\ . \tag{35}$$

Statistics of the above factors are suggested in Table 5.

Table 5 Statistics of random variables involved in the assessment of the wind loading

Variable	Ratio $\frac{\text{Expected}}{\text{Computed}}$	Coefficient of variation, V	Reference
$\overline{Q}_{ref}$	0.8	0.2 - 0.3	Davenport, 1987
c_r	0.8	0.1 - 0.2	
c_a - pressure coefficients	1.0	0.1 - 0.3	
- force coefficients	1.0	0.1 - 0.15	
c_g	1.0	0.1 - 0.15	
c_d	1.0	0.1 - 0.2	
Structure period			Vanmarcke, 1992
- small amplitudes	0.85	0.3 - 0.35	
- large amplitudes	1.15	0.3 - 0.35	
Structure damping			
- small amplitudes	0.8	0.4 - 0.6	
- large amplitudes	1.2	0.4 - 0.6	

Generally, but not necessarily, the lognormal distribution is the recommended probability distribution function for each of the partial factors involved in Eq. (32) and Eq. (34).

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JCSS PROBABILISTIC MODEL CODE
PART 2: LOAD MODELS

2.18 IMPACT LOAD

Table of contents:

List of symbols:

a	=	deceleration
A_b	=	the area of the building including the shadow area
d	=	distance from the structural element to the road
$f_s(y)$	=	distribution of initial object position in y direction
$F_c(x)$	=	static compression strength at a distance x from the nose
k	=	stiffness
m	=	mass
m'(x)	=	mass per unit length
n	=	number of vehicles, ships or planes per time unit
n(t)	=	number of moving objects per time unit (traffic intensity)
P_a	=	the probability that a collision is avoided by human intervention.

List of symbols:

$P_{f\,q}(xy)$	=	the probability of structural failure given a mechanical or human failure on the ship, vehicle, etc. at point (x,y).
r	=	d/sin α = the distance from "leaving point" to "impact point"
R	=	radius of airport influence circle
T	=	period of time under consideration
v_c	=	the object velocity at impact
$v_c(t)$	=	velocity of the crashed part
$v_c\,(xy)$	=	object velocity at impact, given initial failure at point (x,y)
v_o	=	velocity of the vehicle when leaving the track
x,y	=	coordinate system;
α	=	angle between collision course and track direction
$\Lambda(r)$	=	collision rate for crash at distance r from the airport with $r < R$
$\lambda(x,t)$	=	failure intensity as a function of the coordinate x and the time t.

2.18 IMPACT LOAD

2.18.1 Basic Model for Impact Loading

2.18.1.1 Introduction

The basic model for impact loading constitutes of (see figure 2.18.1):

- potentially colliding objects (vehicles, ships, airplanes) that have an intended course, which may be the centre line of a traffic lane, a shipping lane or an air corridor; the moving object will normally have some distance to this centre line;
- the occurrence of a human or mechanical failure that may lead to a deviation of the intended course; these occurences are described by a homogeneous Poison process;
- the course of the object after the initial failure, which depend on both object properties and environment;
- the mechanical impact between object and structure, where the kinetic energy of the colliding object is partly transferred into elastic-plastic deformation or fracture of the structural elements in both the building structure and the colliding object.

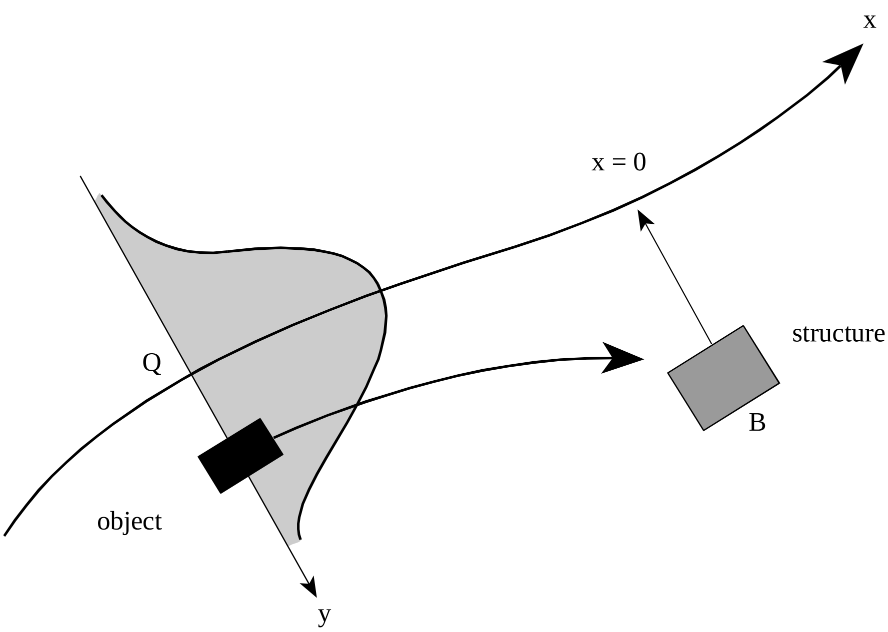


Figure 2.18.1: Probabilistic collision model

2.18.1.2 Failure probability

The probability that a single object, moving in x-direction, suffers from a human or mechanical failure in the square [dx, dy] (see figure 2.1.8.1) and causes collapse at some structure is given by:

$P_{fq}(x,y)\ f_s(y)\ dy\ \lambda(x,t)\ dx$

where:

$f_s(y)$ = distribution of initial object position in y direction (see figure 2.18.1)
$P_{f\,q}(xy)$ = the probability of structural failure given a mechanical or human failure on the ship, vehicle, etc. at point (x,y).
x,y = coordinate system; the x coordinate follows the centre line of the traffic lane, while the y coordinate represents the (horizontal) distance of the object to the centre; the structure that potentially could be hit, is located at the point with coordinates x=0 and y=d.
$\lambda(x,t)$ = failure intensity as a function of the coordinate x and the time t. The length dependency expresses the variability in circumstances along the centre line (for instance curved versus straight trajectories). The time dependency indicates differences in summer and winter, day and night, etc. Note that although $\lambda(x,t)$ is a function of x and t, its dimension is [1/Length].

The probability of structural failure for a period T can then be presented as:

$$P_f(T) = 1 - \exp\{-\int\int\int n(t)\,\lambda(x,t)\,P_{fq}(xy)\,f_s(y)\,dx\,dy\,dt\} \qquad (2.18.1)$$

or for small probability and constant n and λ:

$$P_f(T) = nT\lambda\int\int P_{fq}(x,y)\,f_s(y)\,dy\,dx \qquad (2.18.\ 2)$$

where:

T = period under consideration
n(t) = number of moving objects per time unit (traffic intensity)

2.18.1.3 Distribution function for the impact load

In principle, impact is an interaction phenomenon between the object and the structure. It is not possible to formulate a separate action and a separate resistance function. However, an upper bound for the impact load can be found using the "rigid structure" assumption. If the colliding object is modelled as an elastic single degree of freedom system, with equivalent stiffness k and mass m, the maximum possible resulting interaction force equals:

$$F_c = v_c \sqrt{(km)} \qquad (2.18.3)$$

v_c = the object velocity at impact

Note that (2.18.3) gives the maximum for the external load; dynamic effects within the structure still need to be considered. Note further that simple upperbounds also may be obtained if the structure and or the object behaves plastic: $F_c = \min[F_{ys}, F_{yo}]$ where F_{ys} = yield force of the structure and F_{yo} = yield force of the object; the duration of this load is $\Delta_t = mv_c/F_c$.

Based on formulation (2.18.3) the distribution function for the load F_c can be found:

$$P\{F_C < X\} = 1 - \exp\{-\int\int\int n\,\lambda\, P[v_C(xy)\sqrt{km} > X]\, f_S(y)\, dxdydt\} \quad (2.18.4)$$

$v_c(xy)$ = object velocity at impact, given initial failure at point (x,y)

For small probabilities:

$$P\{F_C > X\} = P_f(T) = nT\lambda \int\int P[v_C\sqrt{km} > X]\, f_S(y)\, dydx \quad (2.18.5)$$

For the designation of the variables, see clause 2.18.1.2.

2.18.2 Impact from vehicles

2.18.2.1 Distribution of impact force

Consider a structural element in the vicinity of a road or track. Impact will occur if some vehicle, travelling over the track, leaves its intended course at some critical place with sufficient speed (see Figure 2.18.2).

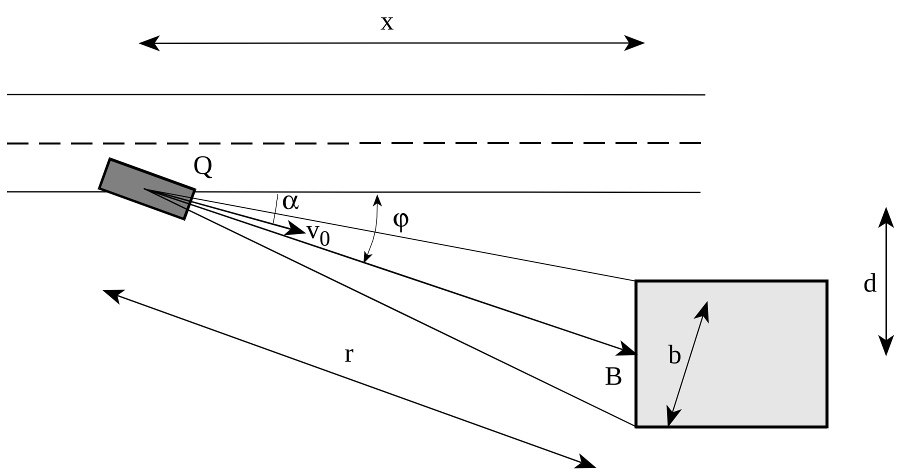


Figure 2.18.2: A vehicle leaves the intended course at point Q with velocity v_0 and angle a. A structural element at distance r is hit with velocity v_r.

The collision force probability distribution based on (2.18.5), neglecting the variability in y-direction is given by:

$$P(F_c > X) = n\,T\,\lambda\,\Delta x\,P\,[\,\sqrt{\{\,m\,k\,(v_o^2 - 2ar)\,\}} > X] \quad (2.18.6)$$

n	= number of vehicles per time unit
T	= period of time under consideration
λ	= probability of a vehicle leaving the road per unit length of track
Δx	= part of the road from where collisions may be expected
v_o	= velocity of the vehicle when leaving the track
a	= deceleration

r = d/sin α = the distance from "leaving point" to "impact point"
d = distance from the structural element to the road
α = angle between collision course and track direction

λ Δx is the probability that a passing vehicle leaves the road at the interval Δx, which is approximated by:

$$\Delta x = b / \sin \mu(\alpha) \tag{2.18.7}$$

The value of b depends on the structural dimensions. However, for small objects such as columns a minimum value of b follows from the width of the vehicle, so b > 2.5 m.

Numerical values and probabilistic models can be found in Table 2.18.1.

variable	designation	type	mean	stand dev
λ	accident rate	deterministic	10^{-10} m^{-1}	-
α	angle of collision course	rayleigh	10 °	10°
v	vehicle velocity - motorway - urban area - court yard - parking garage	 lognormal lognormal lognormal lognormal	 80 km/hr 40 15 10	 10 km/hr 7 6 5
a	deceleration	lognormal	4 m^2/s	1.3 m/s^2
m	vehicle mass - truck - car	 normal normal	 20.000 kg* 1500 kg	 12.000 kg* 400 kg
k	vehicle stiffness	lognormal	300 kN/m	60 kN/m
*Combined with $F = k\sqrt{m}v$ these estimates are quite conservative. One might consider possible reductions due to transformation of energy into rotational movements, etc. e.g. by the concept of “effective mass”				

Table 2.18.1: Numerical values for vehicle impact

2.18.2.2 Specifications of impact force

The collission force is a horizontal force; only the force component perpendicular to the structural surface needs to be considered.

The collision force for passenger cars affects the structure at 0.5 m above the level of the driving surface; for trucks the collision force affects it at 1.25 m above the level of the driving surface. The force application area is 0.25 m (height) times 1.50 m (width).

For impact loads on horizontal structural elements above traffic lanes the following rules hold (see Figure 2.18.3):

a) on vertical surfaces the impact actions follow from 2.18.2.1 and the height reduction as specified at c)
b) on horizontal lower side surfaces upward inclination of 10% should be considered. The force application area is 0.25 m (heigh) times 0.25 m (width).
c) for free heights h larger than 6.0 m the forces are equal to zero; for free heights between 4.0 m and 6.0 m a linear interpolation should be used

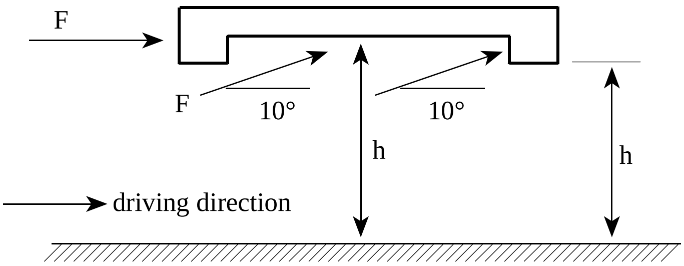


Figure 2.18.3: Impact loads on horizontal structural elements above traffic lanes

2.18.3 Impact from ships

2.18.3.1 Distribution of impact force

A co-ordinate system (x,y) is introduced as indicated in Figure 2.18.4. The x coordinate follows the centre line of the traffic lane, while the y co-ordinate represents the (horizontal) distance of the ship to the centre. The structure that potentially could be hit is located at the point with co-ordinates x=0 and y=d.

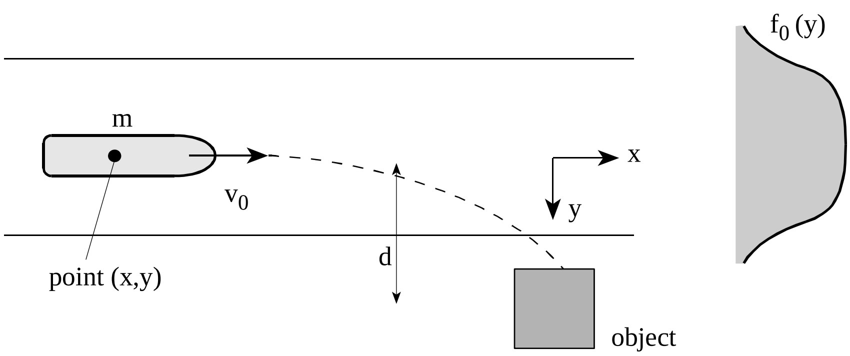


Figure 2.18.4: Ingredients for a ship collision model

Ship impact may be the result of:
(a) either a ship being on collision course, while no avoidance action is taken
(b) a mechanical or human failure leading to a change of course.

In case (a) a ship is on collission course, which is not corrected due to inattendance, bad visibility, old cards and so on. In case (b) the orginal course is correct, but changed, due to e.g. rudder problems or misjudgement.

Both origins (a) and (b) are present in the following model which is a modification of (2.18.1):

$$P(F > X) = n\,T\,P_{na} \int \int_{\Delta y} P[v_c(x,y)\sqrt{km} > X]\, f_s(y)\, dx\, dy$$

$$+\ n\,T\,\lambda \int \int_{-\infty}^{+\infty} P[v_c(x,y)\sqrt{km} > X]\, f_s(y)\, dx\, dy \qquad (2.18.8)$$

T	=	period of time under consideration
n	=	number of ships per time unit (traffic intensity)
λ	=	probability of a failure per unit travelling distance
$v(x,y)$	=	impact velocity of ship, given error at point (x,y)
k	=	stiffness of the ship
m	=	mass of the ship
$f_s(y)$	=	distribution of initial ship position in y direction
P_{na}	=	the probability that a collision is not avoided by human intervention, given collision course
Δy	=	values of y coinciding with a collision course

For the evaluation in practical cases, it may be necessary to evaluate (2.18.8) for various ship types and traffic lanes, and add the results in a proper way at the end of the analysis.

Table 2.18.2 gives a number of standard ship characteristics and velocities that could be chosen by the designer.

variable	designation	type	mean	standard dev
P_{na}	avoidance probability - small - medium - large - very large	-	 0.045 0.003 0.002 0.001	-
λ	failure rate	-	10^{-6} km^{-1}	-
v	velocity - harbour - canal - sea	 lognormal lognormal lognormal	 1.5 m/s 3 6	 0.5 m/s 1.0 1.5
m	mass - small - medium - large - very large	 lognormal lognormal lognormal lognormal	 1000 ton 4000 20000 200000	 2000 ton 8000 40000 200000
k	equivalent stiffness	lognormal	15 MN/m	3 MN/m

Table 2.18.2: Numerical values for ship impact

2.18.3.2 Specifications of impact force

Bow, stern and broad side impact shall be considered where relevant; for side and stern impact the design impact velocities may be reduced.

Bow impact shall be considered for the main sail direction with a maximum deviation of 30^o.

If a wall structure is hit under an angle a, the following forces should be considered:
- perpendicular to the wall: $F_y = F \sin\alpha$
- in wall direction: $F_x = f F \sin\alpha$

where F is the collision force at $\alpha = 90°$ and f = 0.3 is the friction coefficient.

Impact is to be considered as a free horizontal force; the point of impact depends on the geomertry of the structure and the size of the vessel. As a guideline one could take the most unfavourable point ranging from 0.1 L below to 0.1 L above the design water level. The impact area is 0.05 L * 0.1 L unless the stuctural element is smaller.

L is the typical ship length (L = 15, 40, 100 and 300 m for respectively small, medium, large and very large ship size).

The forces on the superstructure of the bridge depend on the height of the bridge and the type of ships te be expected. In general the force on the superstructure of the bridge will be limited by the yield strenght of the ships superstructure. A maximum of 10 000 kN for large and very large ships and 3000 kN for small and medium ships can be taken as a guideline averages.

2.18.4 Impact from airplanes

2.18.4.1 Distribution of impact force

The probability of a structure being hit by an airplane is very small. Only for exceptional structures like nuclear power plants, where the consequences of failure may be very large, is it mandatory to account for aircraft impact during design.

For air corridors, using (2.18.3) and for small probabilities:

$$P(F_c > X) = n\,T\,\lambda\,A_b\,P_{na}\,P(F_c > X|\text{impact})\,f_s(y) \qquad (2.18.9)$$

n	=	number of planes passing per time unit through an air corridor (traffic intensity)
T	=	time period of interest (for instance reference period)
λ	=	probability of a crash per unit distance of flying
$f_s(y)$	=	distribution of ground impact perpendicular to the corridor direction, given a crash
A_b	=	the area of the building including the shadow area
P_{na}	=	probability of not avoiding a collision, given an airplane on collision course

The area A_b is the area of the building itself, enlarged by a so called shadow area (see figure 2.18.5). The strike angle α is random.

For the vicinity of an airport (at a distance r) the impact force distribution is based on:

$$P(F_c > X) = nTP_{na}\,\Lambda(r)A_b\,P\{F_c > X|\text{impact}\} \qquad (2.18.10)$$

$$\Lambda(r) = \frac{\bar{\Lambda}\,R}{2\,r} \qquad (2.18.11)$$

$\bar{\Lambda}$	= average air plane collision rate for a circular area with radius R = 8 km
$\Lambda(r)$	= collision rate for crash at distance r from the airport with r < R
n	= number of planes approaching the airport per windtunnel
R	= radius of airport influence circle
r	= distance to the airport

Numerical values are presented in Table 2.18.3

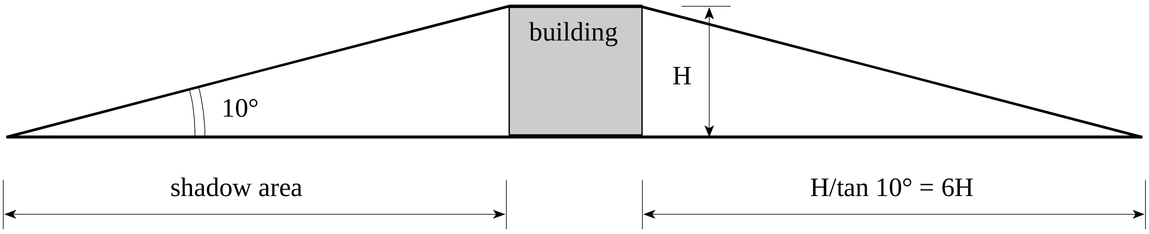


Figure 2.18.5: Strike area A_b for an airplane crash.

For airplanes the impact model (2.18.3) is not sufficient. A better model is given by:

$$F_C(t) = F_C(\xi) + m'(\xi)\, v_C^2(t) \tag{2.18.12}$$

$$\xi = \int_0^t v_C(\tau)\, d\tau \tag{2.18.13}$$

$F_c(x)$ = static compression strength at a distance x from the nose
$m'(x)$ = mass per unit length at a distance x from the nose
$v_c(t)$ = velocity of the crashed part of the plane at time t

Sometimes $v_c(t)$ is taken as constant and equal to v_r for further simplification. Results from calculations based on this model can be found in table (2.18.4).

It is recommended to make the analysis for each type of aircraft (small, large, civil, military) separately and add the results afterwards.

λ	Crash rate - military plane - civil plane	 10^{-8} km^{-1} 10^{-9} km^{-1}
$\overline{\Lambda}$	Average collission rate for airport area - small planes (< 6 ton) - large planes (> 6 ton)	 10^{-4} yr^{-1} km^{-2} 4 10^{-5} yr^{-1} km^{-2}
R	Radius of aiport influence circle	8 km
α	Strike angle	mean 10^o standard deviation 10^o Rayleigh Distribution

Table 2.18.3: Numerical values for the air plane impact model

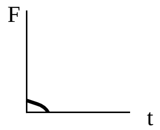
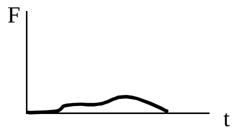
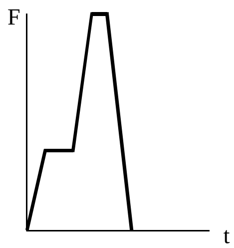
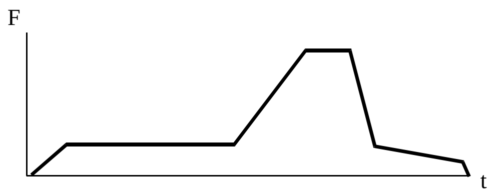
Type	t[ms]	F [MN]	
Cessna 210 A m = 1.7 ton v = 100 m/s A = 7 m^2 engine m = 0.2 ton A = 0.5 m^2	0 3 6 18 18	0 7 7 4 4	
Lear Jet 23 A m = 5.7 ton v = 100 m/s A = 12 m^2	0 20 35 50 70 80 100	0 2 6 6 12 20 0	
MRCA (Multi Role Combat) m = 25 ton v = 215 m/s A = 4 m^2 engine m = 1.2 ton A = 0.5 m^2	0 10 30 40 50 701	0 55 55 154 154 0	
Boeing 707-320 m = 90 ton v = 100 m/s A = 36 m^2	0 30 150 200 230 250 320 330	0 20 20 90 90 20 10 0	

Table 2.18.4: Impact characteristics for various aircrafts (perpendicular on immovable walls

A	=	cross sectional area of the plane or engine
m	=	mass
v_r	=	velocity at impact

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PART 2: LOAD MODELS

2.20 FIRE

Table of contents:

List of symbols:

A_f = floor area
A_i = area of the vertical opening i in the fire compartment [m^2]
A_t = total internal surface area
f = ventilation opening
H_i = specific combustible energy for material i
m_i = derating factor between 0 and 1, describing the degree of combustion
M_{ki} = combustible mass present at ΔA for material i
q_o = fire load density per unit floor area
t = time
t_{eq} = equivalent time of fire duration

α = parameter
β_f = coefficient (model uncertainty)
θ = temperature in the compartment
θ_o = temperature at the start of the fire
θ_A = parameter

2.20. FIRE

2.20.1 Fire ignition model

The probability of a fire starting in a given building or area is modelled as a Poisson process with constant occurrence rate:

$$P\{\text{ignition in } (t,t+dt) \text{ in a compartment}\} = \nu_{fire}\, dt \qquad (2.20.1)$$

The occurrence rate ν_{fire} can be written as a summation of local values over the floor area:

$$\nu_{fire} = \iint_{A_f} \lambda(x,y)\; dxdy \qquad (2.20.2)$$

where $\lambda(x,y)$ corresponds to the probability of fire ignition per year per m^2 for a given occupancy type; A_f is the floor area of the fire compartment. As in most applications $\lambda(x,y)$ can be simplied as a constant, and (2.20.2) can be simplified to:

$$\nu_{fire} = A_f\, \lambda \qquad (2.20.3)$$

Values for l are presented in Table 2.20.1.

Type of building	λ [m^{-2} $year^{-1}$]
dwelling/school	0.5 to 4 * 10^{-6}
shop/office	1 to * 10^{-6}
industrial building	2 to 10 * 10^{-6}

Table 2.20.1: Example values of annual fire probabilities λ per unit floor area for several types of occupancy.

2.20.2 Flashover occurrence

After ignition there are various ways in which a fire can develop. The fire might extinguish itself after a certain period of time because no other combustible material is present. The fire may be detected very early and be extinguished by hand. An automatic sprinkler system may operate or the fire brigade may arrive in time to prevent flash over. Only in a minority of cases does a fire develop into a fully developed room or compartment fire; sometimes the fire may break through a barrier and start a fire in another compartment. From the structural point of view only these fully developed or post flashover fires (see Figure 2.20.1) may lead to failure. For very large fire compartments having a very large concentration of fire loads, e.g. industrial buildings, a local fire of high intensity also may lead to (localised) structural damage.

The occurrence rate of flashover is given by:

$$\nu_{flash\ over} = P\{\text{flash over} \mid \text{ignition}\}\ \nu_{fire} \qquad (2.20.4)$$

The probability of a flashover once a fire has taken place, can obviously be influenced by the presence of sprinklers and fire brigades. Numerical values for the analysis are presented in Table 2.20.2.

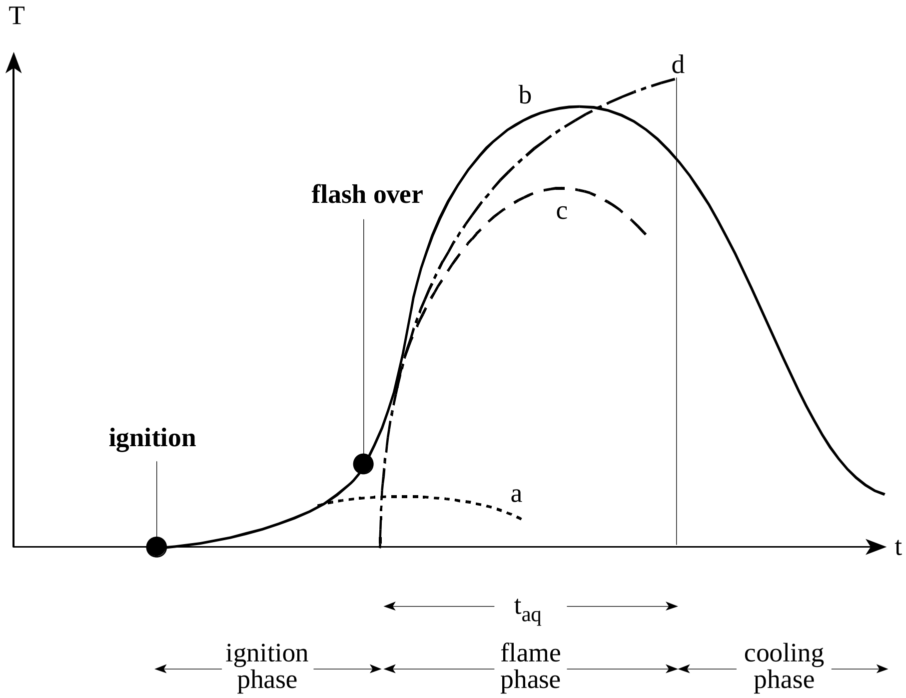

Figure 2.1: Schematic presentation of a temperature-time curve

- * Curve (a) represents the temperature-time curve when a sprinkler system or a timely fire brigade action is successful.
- * Curve (b) presents the temperature-time relation for a fully developed fire.
- * Curve (c) indicates the limited influence of a fire brigade arriving after flashover has taken place.
- * Curve (d) indicates the ISO-standard temperature curve (see section 2.20.4.2).

Protection method	P{flashover|ignition}
Public fire brigade	10^{-1}
Sprinkler	10^{-2}
High standard fire brigade on site, combined with alarm system (industries only)	10^{-3} to 10^{-2}
Both sprinkler and high standard residential fire brigade	10^{-4}

Table 2.20.2: Probability of flashover for given ignition, depending on the type of active protection measures

2.20.3 Combustible material modelling

The available combustible material can be considered as a random field, which in general might be nonhomogeneous as well as nonstationary. The intensity of the field q at some point in space and time is defined as:

$$q = \frac{\Sigma m_i M_{ki} H_i}{A_f} \qquad (2.20.5)$$

m_i = derating factor between 0 and 1, describing the degree of combustion
M_{ki} = combustible mass present at A_f for material i
H_i = specific combustible energy for material i
A_f = considered floor area

In some cases the intensity q may also depend on a vertical ordinate.

The non-dimensional factor μ_i is a function of the fuel type, the geometrical properties of the fuel, and the position of the fuel in the fire compartment, among other things. For some types of fire load components, m_i depends on the time of fire duration and on the gas temperature-time characteristics of the compartment fire. Probabilistic models for q are presented in tabel 2.20.3.

Type of fire compartment	Mean value $\mu(q_0)$ [MJm^{-2}]	Coefficient of variation $V(q_0)$
1 : Dwellings	500	0.20
2 : Offices	600	0.30
3 : Schools	350	0.20
4: Hospitals	450	0.30
5: Hotels	300	0.25

Table 2.20.3: Recommended values for the average fire load intensity q_o

2.20.4 Temperature-time relationship

2.20.4.1 Scientific models

For known characteristics of both the combustible material and the compartment, the post flash over period of the temperature time curve can be calculated from energy and mass balance equations.

Many variables can be introduced as random in the model, for instance:
- the amount and spatial distributions of combustible material;
- the effective energy value;
- the rate of combustion;
- the ventilation characteristics;
- air use and gas production parameters;
- thermal conductivity properties;
- model uncertainties.

In addition, the development of the fire may depend on events like collapse of windows or containments, which may change the ventilation conditions or the available amount of combustible material respectively.

As a simplification the following assumptions may be used.

1. the combustible material is wood;
2. the wood is spread uniformly over the floor area;
3. the fire compartment is of a standard building material (brick, concrete);
4. the fire is controlled by ventilation and not by the amount of fuel load (this is conservative);
5. the initial temperature is 20 °C .

In this case the temperature time curve depends on two parameters:

- the floor averaged fire load density q_o ;
- the opening factor f.

The opening factor f is defined as:

$$f = \frac{A_v}{A_t}\sqrt{h}; \quad \text{with } h = \frac{\sum A_i h_i}{A_v}; \; A_v = \sum A_i \qquad (2.20.7)$$

where:

A_t = total internal surface area of the fire compartment, i.e. the area of the walls, floor and ceiling, including the openings [m^2]
A_i = area of the vertical opening i in the fire compartment [m^2]
h_i = value of the height of opening i [m]

For a fire compartment which also contains horizontal openings, the opening factor can be calculated from a similar expression. In calculating the opening factor, it is assumed that ordinary window glass is immediately destroyed when fire breaks out.

In many cases it will be possible to indicate a physical maximum f_{max}. The actual value of f in a fire should be modelled as a random quantity according to:

$$f = f_{max}(1 - \zeta) \qquad (2.20.8)$$

ζ = random parameter (see Table 2.20.4)

To avoid negative values of f, this lognormal distribution should be cut off at $\zeta = 1$. In addition one should multiply the resulting temperatures by an overall model uncertainty factor θ_{model}.

2.20.4.2 Engineering models

In many engineering applications, use is made of equivalent standard temperature-time-relationship according to ISO 834:

$\theta = \theta_o + \theta_A \log_{10} \{\alpha t + 1\}$ for $0 < t < t_{eq}$ (2.20.9)

with:

$$t_{eq} = \frac{\beta_f \; q_o \; A_f}{A_t \; \sqrt{f}} \qquad (2.20.10)$$

θ = temperature in the compartment
θ_o = temperature at the start of the fire
θ_A = parameter
α = parameter
t = time
t_{eq} = equivalent time of fire duration
β_f = coefficient (model uncertainty)
q_o = fire load density per unit floor area
A_f = floor area
A_t = total internal surface area
f = ventilation opening (see 2.20.7, 2.20.8)

Numerical values and probabilistic models are given in Table 2.20.4.

Variable	Distribution	Mean	Standard deviation
ζ	truncated lognormal [1)]	0.2	0.2
β_f	lognormal	4.0 $sm^{2.25}$/MJ	1.0
θ_o	deterministic	20^oC	-
θ_A	deterministic	345 K	-
α	deterministic	0.13 s^{-1}	-

[1)] values of $\zeta > 1$ should be supressed

Table 2.20.4: Numerical values for random variables